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# Love for Quality, Comparative Advantage, and Trade\*

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## Abstract

We propose a Ricardian trade model with horizontal and vertical differentiation, where individuals' willingness to pay for quality rises with their income, and productivity differentials across countries are stronger for high-quality goods. Our theory predicts that the scope for trade widens and international specialisation intensifies as incomes grow and wealthier consumers raise the quality of their consumption baskets. This implies that comparative advantages intensify gradually over the path of development as a by-product of the process of quality upgrading. The evolution of comparative advantages leads to specific trade patterns that change over the growth path, by linking richer importers to more specialised exporters. We provide empirical support for this prediction, showing that the share of imports originating from exporters exhibiting a comparative advantage in a specific product correlates positively with the importer's GDP per head.

**Keywords:** International Trade, Nonhomothetic Preferences, Quality Ladders.

**JEL Classifications:** F11, F43, O40

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# 1 Introduction

Income is a key determinant of consumer choice. A crucial dimension through which purchasing power influences this choice is the quality of consumption. People with very different incomes tend to consume commodities within the same category of goods, such as clothes, cars, wines, etc. However, the actual quality of the consumed commodities differs substantially when looking at poorer versus wealthier households. The same reasoning naturally extends to countries with different levels of income per capita. In this case, the quality dimension of consumption entails important implications on the evolution of trade flows.

Several recent studies have investigated the links between quality of consumption and international trade. One strand of literature has centred their attention on the demand side, finding a strong positive correlation between quality of imports and the importer's income per head [Hallak (2006), Fieler (2011a)].<sup>1</sup> On the other hand, a set of papers have focused on whether exporters adjust the quality of their production to serve markets with different income levels. The evidence here also points towards the presence of nonhomothetic preferences along the quality dimension, showing that producers sell higher quality versions of their output to richer importers.<sup>2</sup>

These empirical findings have motivated a number of models that yield trade patterns where richer importers buy high-quality versions of goods, while exporters differentiate the quality of their output by income at destination [Hallak (2010), Fajgelbaum, Grossman and Helpman (2011), Jaimovich and Merella (2012)]. Yet, this literature has approached the determinants of countries' sectoral specialisation as a phenomenon that is independent of the process of quality upgrading resulting from higher consumer incomes. This paper investigates whether exchanging higher or lower quality versions of output affects the categories of goods that countries specialise in, and the intensity of trade links that they establish with different importers. We propose a theory where quality upgrading in consumption becomes the central driving force behind a general process of comparative advantage intensification and varying bilateral links at different

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<sup>1</sup>See also related evidence in Choi, Hummels and Xiang (2009), Francois and Kaplan (1996) and Dalgin, Trindade and Mitra (2008).

<sup>2</sup>For example, Verhoogen (2008) and Iacovone and Javorcik (2008) provide evidence of Mexican manufacturing plants selling higher qualities in US than in their local markets. Brooks (2006) establishes the same results for Colombian manufacturing plants, and Manova and Zhang (2011) show that Chinese firms ship higher qualities of their exports to richer importers. Analogous evidence is provided by Bastos and Silva (2010) for Portuguese firms, and by Crino and Epifani (2012) for Italian ones.

levels of income.

Our theory is grounded on the hypothesis that productivity differentials are stronger for higher-quality goods, combined with the notion that willingness to pay for quality rises with income. Within this framework, we show that international specialisation and trade intensify over the growth path. The evolution of trade flows featured by our model presents novel specificities that stem from the interaction between nonhomothetic preferences and sectoral productivity differentials that become more pronounced at higher levels of quality. In particular, the process of quality upgrading with rising income sets in motion both demand-driven and supply-driven factors that leads to a simultaneous rise in specialisation by importers and exporters over the growth path. Import and export specialisation take place together precisely because, as countries become richer, consumers shift their spending towards high-quality goods, which are exactly those that tend to display greater scope for export specialisation.

We model a world economy with a continuum of horizontally differentiated goods, each of them available in a continuum of vertically ordered quality levels. Each country produces a particular variety of every good. The production technology differs both across countries and sectors, with some countries being intrinsically better than others in producing certain types of goods. This represents the traditional source of trade in Ricardian models, and leads to specialisation along the horizontal dimension of the commodity space. Alongside this traditional feature, we assume that intrinsic productivity differentials (on the horizontal dimension) tend to become increasingly pronounced as production moves up (vertically) on the quality ladders of each good. In other words, a country may have, for example, a cost advantage in producing wine, while another country may have it in whisky. This would naturally lead them to exchange these two goods. Yet, in our model, productivity differences in the wine and whisky industries do not remain constant along the quality space, but become more intense as production moves up towards higher quality versions of those goods. As a result, the scope for international trade turns out to be wider for high-quality wines and whiskies than for low-quality ones.

We combine such a production structure with nonhomothetic preferences on the quality dimension. This implies that, given market prices, richer individuals consume higher quality versions of the horizontally differentiated goods. Within this framework, we show that at low levels of income both export and import specialisation remain low. The reason for this result is that productivity differentials across producers from different economies are relatively narrow for goods offered in low quality versions. However, in a growth context, as individuals upgrade their quality of consumption, sectoral productivity differentials become wider, which in turn

leads to a gradual process of increasing international specialisation.

Our model thus suggests that the study of the evolution of trade links may require considering a more flexible concept of comparative advantage than the one traditionally used in the literature, so as to encompass quality upgrading as an inherent part of it. In the literature of Ricardian trade, the comparative advantage is solely determined by exporters' technologies. This paper instead sustains that both the importers' incomes and the exporters' sectoral productivities must be taken into account in order to establish a rank of comparative advantage. This is because the degree of comparative advantage between any two countries is crucially affected by the quality of consumption of their consumers. As a consequence, richer and poorer importers may end up establishing trade links with different partners, simply because the gaps between their willingness-to-pay for quality may translate into unequal degrees of comparative advantage with respect to the same set of exporters.

The conditionality of comparative advantage on importers incomes entails clear and testable predictions on the evolution of trade flows in a growth context. In particular, the model yields predictions that link different importers to specific exporters. According to our model, the share of imports originating from exporters exhibiting an advantage in producing a given good must grow with the income per head of the importer. This would be the result of richer importers buying high-quality versions of goods, which are the type of commodities for which cost differentials across countries are relatively more pronounced. In that regard, we first test the notion that productivity differentials become more intense at higher levels of quality of production. Next, we provide evidence consistent with the prediction that richer economies are more likely to buy their imports from producers who display a comparative advantage in the imported goods.

## Related Literature

Nonhomothetic preferences are by now a common modelling choice in the trade literature. However, most of the past trade literature with nonhomotheticities has focused either on vertical differentiation [*e.g.*, Flam and Helpman (1987), Stokey (1991) and Murphy and Shleifer (1997)] or horizontal differentiation in consumption [*e.g.*, Markusen (1986), Bergstrand (1990) and Matsuyama (2000)].<sup>3</sup> Two recent articles have combined vertical and horizontal differentiation with

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<sup>3</sup>For some recent contributions with horizontal differentiation and nonhomothetic preferences see, for example: Foellmi, Hopenstrick and Zweimuller (2010) and Tarasov (2011), where consumers are subject to a discrete consumption choice (they must consume either zero or one unit for each good), and Fieler (2011b) who, using a CES utility function, ties the income elasticity of consumption goods across different industries to the degree of substitution of goods within the same industry.

preferences featuring income-dependent willingness to pay for quality: Fajgelbaum, Grossman and Helpman (2011) and Jaimovich and Merella (2012).

Fajgelbaum et al. (2011) analyse how differences in income distributions between economies with access to the *same* technologies determine trade flows in the presence of increasing returns and trade cost. Like ours, their paper leads to an endogenous emergence of comparative advantages, which may have remained latent for quite some time (either due to trade costs being too high or countries' income distributions being too similar). Our paper, instead, sticks to the Ricardian tradition where trade is the result of *differences* in technologies featuring constant returns to scale. In particular, in our model, comparative advantages and trade emerge gradually, not because trade costs obstruct the course of increasing returns, but because the demand for commodities displaying wider heterogeneity in cost of production (*i.e.*, high-quality goods) expands as incomes rise. In that respect, a fundamental difference between the two papers is that in Fajgelbaum et al. (2011) there is no particular reason why high-quality versions of goods are inherently more tradable than low-quality ones (tradeability and specialisation there are essentially determined by differences in the income distributions of countries).

Jaimovich and Merella (2012) also propose a nonhomothetic preference specification where budget reallocations take place both within and across horizontally differentiated goods. That paper, however, remained within a standard Ricardian framework where absolute and comparative advantages are determined from the outset, and purely by technological conditions. Hence, nonhomothetic preferences play no essential role there in determining export and import specialisation at different levels of development. By contrast, it is the interaction between rising differences in productivity at higher quality levels and nonhomotheticities in quality that generates our novel results in terms of co-evolution of export and import specialisation.

A key assumption in our theory is the widening in productivity differentials at higher levels of quality. To the best of our knowledge, Alcalá (2011) is the only other paper that has explicitly introduced a similar feature into a Ricardian model of trade. An important difference between the two papers is that Alcalá's keeps the homothetic demand structure presented in Dornbusch, Fisher and Samuelson (1977) essentially intact. Nonhomotheticities in demand are actually crucial to our story and, in particular, to its main predictions regarding the evolution of trade flows and specialisation at different levels of income.

Finally, Fieler (2011b) also studies the interplay between nonhomothetic demand and Ricardian technological disparities. She shows that, when productivity differences are stronger for goods with high income elasticity, her model matches quite closely key features of North-North

and North-South trade. Our model differs from hers in that the effects of demand on trade stem from the allocation of spending *within* categories of goods rather than *across* them.<sup>4</sup> Our results therefore hinge on richer consumers switching their good-specific expenditure shares from lower-quality to higher-quality versions of the goods. It is in fact this *within-good* substitution process that leads to our predictions of income-dependent spending shares across different exporters.<sup>5</sup>

The rest of the paper is organised as follows. Section 2 studies a world economy with a continuum of countries where all economies have the same level of income per head in equilibrium. Section 3 generalises the main results to a world economy where some countries are richer than others. Section 4 presents some empirical results consistent with the main predictions of our model. Section 5 concludes. All relevant proofs can be found in the Appendices.

## 2 A world economy with equally rich countries

We study a world economy with a unit continuum of countries indexed by  $v \in \mathbb{V}$ . In each country there is a continuum of individuals with unit mass. Each individual is endowed with one unit of labour time. We assume labour is immobile across countries. In addition, we assume all countries are open to international trade, and there are no trading costs of any sort.

### 2.1 Commodity space and production technologies

All countries share a common commodity space defined along three distinct dimensions: a *horizontal*, a *varietal*, and a *vertical* dimension.

Concerning the horizontal dimension, there exists a unit continuum of differentiated goods, indexed by  $z$ , where  $z \in \mathbb{Z} = [0, 1]$ . In terms of the varietal dimension, we assume that each country  $v \in \mathbb{V}$  produces a particular *variety*  $v$  of each differentiated good  $z$ . Finally, our vertical dimension refers to the intrinsic quality of the commodity: a continuum of different qualities  $q$ , where  $q \in \mathbb{Q} = [1, \infty)$ , are potentially available for every variety  $v$  of each good  $z$ . As a result, in our setup, each commodity is designated by a specific good-variety-quality index,  $(z, v, q) \in \mathbb{Z} \times \mathbb{V} \times \mathbb{Q}$ .<sup>6</sup>

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<sup>4</sup>This sort of adjustment is absent in her model, as goods there differ only on the horizontal dimension.

<sup>5</sup>In this respect, our paper relates also to Linder (1961) and Hallak (2010) views of quality as an important dimension in explaining trade flows between countries of similar income levels. We propose a new mechanism that links together quality of production, income per capita and trade at different stages of development.

<sup>6</sup>To fix ideas, the horizontal dimension refers to different types of goods, such as cars, wines, coffee beans, etc.

In each country  $v$  there exists a continuum of firms that may transform local labour into commodities of variety  $v$ . Production technologies are idiosyncratic both to the sector  $z$  and to the country  $v$ . In particular, we assume that, in order to produce one unit of commodity  $(z, v, q)$ , a firm from country  $v$  in sector  $z$  needs to use  $\Gamma_{z,v}(q)$  units of labour, where:

$$\Gamma_{z,v}(q) = e^{-\eta_{z,v}} \frac{q^{\eta_{z,v}}}{1 + \kappa}, \quad \text{with } \kappa > 0.$$

Unit labour requirements contain two key technological parameters. The first is  $\kappa$ , which applies identically to all sectors and countries, and we interpret it as the worldwide total factor productivity level.<sup>7</sup> The second is  $\eta_{z,v}$ , which may differ both across  $z$  and  $v$ , and governs the elasticity of the labour requirements with respect to quality upgrading.<sup>8</sup>

In what follows, we assume that each parameter  $\eta_{z,v}$  is *independently* drawn from a probability density function with uniform distribution over the interval  $[\underline{\eta}, \bar{\eta}]$ . In addition, we assume that  $\underline{\eta} > 1$ . Hence, the labour requirement functions  $\Gamma_{z,v}(q)$  are always strictly increasing and convex in  $q$ .

Henceforth, we denote by  $w_v$  the wage per unit of labour time in country  $v$ . Since in each sector  $z$  of each country  $v$  there is a continuum of firms that are able to produce commodity  $(z, v, q)$  at identical unit cost, in equilibrium, all commodities in the world will be priced exactly at their unit cost. That is,

$$p_{z,v,q} = e^{-\eta_{z,v}} \frac{q^{\eta_{z,v}}}{1 + \kappa} w_v, \quad \text{for all } (z, v, q) \in \mathbb{Z} \times \mathbb{V} \times \mathbb{Q}. \quad (1)$$

## 2.2 Preferences and budget constraint

All individuals in the world share identical preferences defined over the good-variety-quality space described in the previous subsection.

We assume that individuals consume *only* one quality  $q \in \mathbb{Q}$  of each variety  $v$  of good  $z$ . Henceforth, to ease notation, we denote the *consumed* quality of variety  $v$  of good  $z$  simply

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The varietal dimension refers to the different varieties of a given type of good, originating from different countries, such as Spanish and French wines (differing in specific traits like the types of grapes and regional vinification techniques). The vertical dimension refers to the intrinsic quality of each specific commodity (*e.g.*, the ageing and the grapes selection in the winemaking).

<sup>7</sup>As such, in our model, increases in  $\kappa$  will capture the effects of aggregate growth and rising real incomes.

<sup>8</sup>The multiplicative term  $e^{-\eta_{z,v}}$  is just included to help simplifying the algebra of the consumer's optimisation problem. All our results still hold true when the unit labour requirements are given by  $\Gamma_{z,v}(q) = q^{\eta_{z,v}} / (1 + \kappa)$ , only at the cost of more tedious algebra.



by  $q_{z,v}$ . Also, we denote by  $c_{z,v} \in \mathbb{R}_+$  the consumed quantity of commodity  $q_{z,v}$ . Individual's preferences are summarised by the following utility function:

$$U = \int_{\mathbb{Z}} \int_{\mathbb{V}} \ln C_{z,v} dv dz, \quad (2)$$

$$\text{with } C_{z,v} = \begin{cases} c_{z,v} & \text{if } c_{z,v} < 1, \\ (c_{z,v})^{q_{z,v}} & \text{if } c_{z,v} \geq 1. \end{cases} \quad (3)$$

Utility  $U$  in (2) is defined as an additive-separable logarithmic function of the (variety-specific) quality-adjusted consumption indices  $C_{z,v}$  defined in (3).<sup>9</sup> Notice from (3) that  $C_{z,v}$  is non-decreasing in  $q_{z,v}$ , hence quality is always a desirable feature. Moreover, (3) implies that the quality dimension becomes increasingly desirable as physical consumption rises, since  $q_{z,v}$  only magnifies the utility derived from (physical) consumption when  $c_{z,v} > 1$ .

An individual with income  $w$  will maximise (2) subject to the following budget constraint:

$$\int_{\mathbb{Z}} \left[ \int_{\mathbb{V}} \left( e^{-\eta_{z,v}} \frac{(q_{z,v})^{\eta_{z,v}}}{1 + \kappa} w_v \right) c_{z,v} dv \right] dz \leq w, \quad (4)$$

where we have already substituted the price  $p_{z,v}$  of each consumed commodity  $q_{z,v}$  by its expression as a function of technological parameters and wage according to (1).

### 2.3 Utility maximisation

When optimising (2) subject to (4) we must take into account the fact that the consumer's income may well differ across countries. Hereafter, we use the letter  $i \in \mathbb{V}$  to refer to the country of origin of a specific consumer, and we use  $i$  as a *superindex* any time we refer to *choices* made by consumers from country  $i$ .

The consumer's problem then requires choosing combinations of (non-negative) quantities on the good-variety-quality (three-dimensional) commodity space, subject to (4). However, it turns out that the optimisation problem may be greatly simplified by making use of the following two intermediate results.

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<sup>9</sup>The additive-separable logarithmic functional form in (2) is posed to ease the exposition. The model delivers qualitatively identical results under a more general utility function that allows for non-unit degrees of substitution between the different commodities  $(z, v) \in \mathbb{Z} \times \mathbb{V}$ . Furthermore, the assumption of a single consumed quality for each good is also posed to ease exposition, and it corresponds to the solution that arises when assuming an *infinite* degree of substitution between qualities of the *same* commodity  $(z, v) \in \mathbb{Z} \times \mathbb{V}$ . More precisely, our results hold qualitatively intact, only at the cost of more tedious algebra, if we replace (2) with the following alternative function:

$$U = \left\{ \int_{\mathbb{Z}} \left[ \int_{\mathbb{V}} \left( \int_{\mathbb{Q}} C_{z,v}(q) dq \right)^{\rho} dv \right]^{\sigma/\rho} dz \right\}^{1/\sigma}, \text{ with } \rho, \sigma \in (0, 1].$$

First, we may note that the functional form in (3) implies consumers would choose  $q_{z,v}^i > 1$  only if  $c_{z,v}^i > 1$  (otherwise they simply set  $q_{z,v}^i = 1$ ).<sup>10</sup> As a result, without any loss of generality, we may simply replace the quality-adjusted consumption indices  $C_{z,v}^i$  in (2) by  $C_{z,v}^i = (c_{z,v}^i)^{q_{z,v}^i}$ .

Second, by letting  $\beta_{z,v}^i$  denote henceforth the demand intensity in country  $i \in \mathbb{V}$  for the variety  $v \in \mathbb{V}$  of good  $z \in \mathbb{Z}$ , we may note that  $c_{z,v}^i = \beta_{z,v}^i w_i / p_{z,v}$  (where recall that  $p_{z,v}$  is the market price of commodity  $q_{z,v}$ ). Hence, using (1), we may write:

$$c_{z,v}^i = \frac{\beta_{z,v}^i w_i}{e^{-\eta_{z,v}} (q_{z,v}^i)^{\eta_{z,v}} w_v / (1 + \kappa)}. \quad (5)$$

The previous two results allow us to restate the original consumer's optimisation problem into a much simpler one, defined only in terms of *optimal consumed qualities* and *optimal budget allocations* across varieties of goods. Below we state the *reformulated* consumer's problem.

An individual from country  $i \in \mathbb{V}$  chooses the optimal quality  $q_{z,v}^i$  and optimal budget allocation  $\beta_{z,v}^i$  for each commodity  $(z, v) \in \mathbb{Z} \times \mathbb{V}$ , so as to solve:<sup>11</sup>

$$\begin{aligned} \max_{\{q_{z,v}^i, \beta_{z,v}^i\}_{(z,v) \in \mathbb{Z} \times \mathbb{V}}} \quad & U = \int_{\mathbb{Z}} \left[ \int_{\mathbb{V}} q_{z,v}^i \left[ \ln \left( \frac{(1 + \kappa) \beta_{z,v}^i w_i}{(q_{z,v}^i)^{\eta_{z,v}} w_v} \right) + \eta_{z,v} \right] dv \right] dz \\ \text{subject to:} \quad & \int_{\mathbb{Z}} \int_{\mathbb{V}} \beta_{z,v}^i dv dz = 1, \quad \text{and} \quad q_{z,v}^i \in \mathbb{Q}, \quad \text{for all } (z, v) \in \mathbb{Z} \times \mathbb{V}. \end{aligned} \quad (6)$$

We can observe that relative wages ( $w_i/w_v$ ) may play a role in the optimisation problem (6). For the time being, we will shut down this channel, and characterise the solution of (6) only for the case in which wages are the same in all countries. (In any case, as it will be shown next, in this specification of the model all wages will turn out to be equal in equilibrium.)

**Lemma 1** *When  $w_v = w$  for all  $v \in \mathbb{V}$ , problem (6) yields, for all  $(z', v') \in \mathbb{Z} \times \mathbb{V}$ :*

$$q_{z',v'}^i = \left( \frac{1 + \kappa}{\int_{\mathbb{Z}} \int_{\mathbb{V}} q_{z,v}^i dv dz} \right)^{1/(\eta_{z',v'} - 1)}, \quad (7)$$

$$\beta_{z',v'}^i = \frac{(1 + \kappa)^{1/(\eta_{z',v'} - 1)}}{(\int_{\mathbb{Z}} \int_{\mathbb{V}} q_{z,v}^i dv dz)^{\eta_{z',v'}/(\eta_{z',v'} - 1)}}. \quad (8)$$

In addition,  $\partial q_{z',v'}^i / \partial \kappa > 0$ .

**Proof.** In Appendix A. ■

<sup>10</sup>This result follows immediately from (3), which implies that  $q_{z,v}$  only magnifies the effect of  $c_{z,v}$  when  $c_{z,v} > 1$  while, according to (1), the price  $p_{z,v}$  is strictly increasing along the quality space.

<sup>11</sup>A formal solution of problem (6) is provided in Appendix A.

Lemma 1 characterises the solution of the consumer's optimisation problem in terms of two sets of variables: (i) the expressions in (7), which stipulate the quality level in which each variety of every horizontally differentiated good is optimally consumed; (ii) the expressions in (8) describing the optimal expenditure shares allocated to those commodities. Furthermore, the result  $\partial q_{z,v}^i / \partial \kappa > 0$  summarises the key nonhomothetic behaviour present in our model: quality upgrading of consumption. That is, the fact that consumers upgrade the quality of all goods they consume as their real incomes grow.

## 2.4 Equilibrium and specialisation

In equilibrium, total world spending on commodities produced in country  $v$  must equal the total labour income in country  $v$  (which is itself equal to the total value of goods produced in  $v$ ). Bearing in mind (6), we may then write down the market clearing conditions as follows:

$$\int_{\mathbb{Z}} \int_{\mathbb{V}} \beta_{z,v}^i w_i di dz = w_v, \quad \text{for all } v \in \mathbb{V}. \quad (9)$$

More formally, an equilibrium in the world economy is given by a set of wages  $\{w_v\}_{v \in \mathbb{V}}$  such that: *i*) prices of all traded commodities are determined by (1); *ii*) consumers from country  $i \in \mathbb{V}$  choose their allocations  $\{q_{z,v}^i, \beta_{z,v}^i\}_{(z,v) \in \mathbb{Z} \times \mathbb{V}}$  by solving (6); and *iii*) the market clearing conditions stipulated in (9) hold simultaneously for all countries.

**Proposition 1** *Suppose that, for each commodity  $(z, v) \in \mathbb{Z} \times \mathbb{V}$ ,  $\eta_{z,v}$  is independently drawn from a uniform density function with support  $[\underline{\eta}, \bar{\eta}]$ . Then, for any  $\kappa > 0$ , in equilibrium:  $w_v = w$  for all  $v \in \mathbb{V}$ .*

**Proof.** *In Appendix A. ■*

Proposition 1 shows that, in this (symmetric) world economy, the equilibrium relative wages remain unchanged and equal to unity all along the growth path. The reason for this result is the following: as  $\kappa$  rises, and real incomes accordingly increase, aggregate demands and supplies grow together at identical speed in all countries. As a consequence, markets clearing conditions in (9) will constantly hold true without the need of any adjustment in relative wages across economies.

The equiproportional *aggregate* variations implicit in Proposition 1 conceal the fact that, as  $\kappa$  increases, economies actually experience significant changes in their productive structure at the *sectoral* level. In other words, although aggregate demands and supplies change at the same speed in all countries, sectoral demands and supplies do not, which in turn leads to

country-specific processes of labour reallocation across sectors. Such sectoral reallocations of labour stem from the interplay of demand and supply side factors. On the demand side, as real incomes grow with a rising  $\kappa$ , individuals start consuming higher quality varieties of each commodity – as can be observed from (7). On the supply side, heterogeneities in sectoral labour productivities across countries become stronger as producers intend to raise the quality of their output. Hence, the interplay between income-dependent willingness to pay for quality and the intensification of sectoral productivity differences at higher levels of quality leads to a process of increasing sectoral specialisation as  $\kappa$  rises.

In what follows we study the effects of the above-mentioned sectoral reallocations of labour on the trade flows across economies. In particular, we focus on the evolution of the revealed comparative advantage (RCA) of every country  $v \in \mathbb{V}$  in each differentiated good  $z \in \mathbb{Z}$ , as we let the worldwide total factor productivity parameter  $\kappa$  rise.

For every commodity  $(z, v)$ , we thus compute the ratios:

$$RCA_{z,v} \equiv \frac{X_{z,v}/X_v}{W_z/W}; \quad (10)$$

where  $X_{z,v}$  (resp.  $W_z$ ) is the total value of exports of good  $z$  by country  $v$  (resp. by the world), and  $X_v$  (resp.  $W$ ) is the aggregate value of exports by country  $v$  (resp. by the world).

Since in our model each country sells a negligible share of its own production domestically, we can safely disregard the effect of sales to local consumers and simply write:

$$X_{z,v} = \int_{\mathbb{V}} \beta_{z,v}^i di \quad \text{and} \quad X_v = \int_{\mathbb{Z}} \int_{\mathbb{V}} \beta_{z,v}^i di dz.$$

In addition:

$$W_z \equiv \int_{\mathbb{V}} X_{z,v} dv \quad \text{and} \quad W \equiv \int_{\mathbb{Z}} W_z dz.$$

Consider first the variables relative to country  $v$ . We can observe that Proposition 1 implies that  $\beta_{z,v}^i = \beta_{z,v}$  for all  $i \in \mathbb{V}$ . Hence, since  $\mathbb{V}$  has unit measure,  $X_{z,v} = \beta_{z,v}$ . Moreover, from Proposition 1 and (9), it follows that  $\int_{\mathbb{Z}} \beta_{z,v} dz = 1$ . Therefore,  $X_v = 1$ .

Let us look now at the world-level variables,  $W_z$  and  $W$ . Notice that, by the law of large numbers, when considering country-specific draws, for every good  $z \in \mathbb{Z}$ , the sequence of sectoral productivity draws  $\{\eta_{z,v}\}_{v \in \mathbb{V}}$  will turn out to be uniformly distributed over the interval  $[\underline{\eta}, \bar{\eta}]$  along the countries space  $\mathbb{V}$ . As a consequence, the world spending on good  $z$  will be equal for all goods, in turn implying that  $W_z = \int_{\mathbb{V}} \beta_{z,v} dv = 1$  for all  $z \in \mathbb{Z}$ .<sup>12</sup> Furthermore, since  $W \equiv \int_{\mathbb{Z}} W_z dz$ , we also have that  $W = 1$ .

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<sup>12</sup>See the proof of Proposition 1 for a formal discussion of this argument.

Plugging all these results into (10) finally implies that:

$$RCA_{z,v} = \beta_{z,v}. \quad (11)$$

In other words, the revealed comparative advantage of country  $v$  in good  $z$ , which represents our indicator for the degree of export specialisation, is given by the total value of exports of good  $z$  by country  $v$ . In our symmetric world economy, the total value of exports equals the demand intensity for commodity  $(z, v)$ , which is identical to all countries  $i \in \mathbb{V}$  and may in turn be interpreted as a measure of the degree of import specialisation.

The following proposition characterises in further detail the main properties of each  $\beta_{z,v} \in \mathbb{Z} \times \mathbb{V}$  in this world economy with identical incomes for all countries. Subsequently, we provide some economic interpretation of the formal results in Proposition 2 in terms of both exports and imports specialisation.

**Proposition 2** *In a symmetric world economy, the value of  $\beta_{z,v}$  equals both: (a) the demand intensity for commodity  $(z, v)$  by any country in  $\mathbb{V}$ ; and (b) the RCA of country  $v$  in good  $z$ . For any pair of commodities  $(z', v'), (z'', v'') \in \mathbb{Z} \times \mathbb{V}$ , with  $\eta_{z',v'} < \eta_{z'',v''}$ , the values of  $\beta_{z',v'}$  and  $\beta_{z'',v''}$  satisfy the following properties:*

$$(i) \beta_{z',v'} > \beta_{z'',v''}.$$

$$(ii) \partial \beta_{z',v'} / \partial \kappa > \partial \beta_{z'',v''} / \partial \kappa.$$

*In addition, defining  $\hat{\eta} \equiv \int_{\mathbb{Z}} \int_{\mathbb{V}} \eta_{z,v} \phi_{z,v} dv dz / \int_{\mathbb{Z}} \int_{\mathbb{V}} \phi_{z,v} dv dz$ , with  $\phi_{z,v} \equiv q_{z,v} / (\eta_{z,v} - 1)$ , for  $\eta_{z',v'} < \hat{\eta} < \eta_{z'',v''}$ :*

$$(iii) \partial \beta_{z'',v''} / \partial \kappa < 0 \text{ and } \partial \beta_{z',v'} / \partial \kappa > 0.$$

**Proof.** *In Appendix A. ■*

The results collected in Proposition 2 characterise the link between sectoral productivities and labour allocations across sectors. Part (i) states that larger shares of resources are allocated to sectors that received better productivity draws (i.e., sectors carrying lower  $\eta_{z,v}$ ). Next, part (ii) of the proposition establishes that the concentration of resources towards those sectors further intensifies as world incomes rise. Finally, part (iii) shows that there exists a threshold  $\hat{\eta}$ , such that sectors whose  $\eta_{z',v'} < \hat{\eta}$  experience an increase in their shares when  $\kappa$  grows, while the opposite occurs to sectors whose  $\eta_{z'',v''} > \hat{\eta}$ .

From a supply side perspective, Proposition 2 allows two types of interpretations. Firstly, by fixing  $v'' = v'$ , we can compare different sectors of a given exporter. From this perspective, the

proposition states that countries export more from those sectors where they enjoy higher labour productivity and a stronger RCA. Secondly, by fixing  $z'' = z'$ , we may compare a given sector across different exporters. In this case, recalling (11), we can observe the RCA of exporter  $v$  in sector  $z$  turns out to be monotonically linked to the productivity draw  $\eta_{z,v}$ : that is, countries that receive better draws for sector  $z$  enjoy a stronger revealed comparative advantage in that sector. In addition, notice that, according to part (ii) of the proposition, both sectoral specialisation and export specialisation intensify as  $\kappa$  increases over the growth path.

From a demand side perspective, part (ii) of Proposition 2 may be interpreted as a result on increasing import specialisation along the growth path. In particular, from the importer's perspective (and fixing  $z'' = z'$ ), our model predicts that as economies get richer, for each good  $z \in \mathbb{Z}$ , we observe a process of growing expenditure shares allocated to the varieties of  $z$  produced by exporters who received better productivity draws in that sector.

The equilibrium characterised in this section has the particular feature that revealed comparative advantages coincide with demand intensities. This is clearly a very specific result that hinges on the assumed symmetry in the distributions of sector-specific productivities across countries. The next section shows that this is no longer the case when we introduce some asymmetry across countries. As we will see, although the results discussed here hold qualitatively unchanged, an asymmetric world leads to a richer characterisation of the links between export specialisation, import specialisation and income per capita.

### 3 A world economy with cross-country inequality

The previous section has dealt with a world economy where, in equilibrium, all countries exhibit the same real income. In this section, we slightly modify the previous setup in order to give room for cross-country inequality. On one side, this extension allows us to generalise the previous results concerning export specialisation to a case in which productivity differentials and cost differentials may *not* always coincide, as a result of equilibrium wages that are different between some countries. On the other side, introducing cross-country inequality allows us to generate more powerful predictions concerning import specialisation (in terms of export sources) at different income levels, which we will later on contrast with the data in Section 4.

In what follows we keep the same commodity space and preference structure as those previously used in Section 2. However, we now assume that the world  $\mathbb{V}$  is composed by two subsets of countries, each with positive measure. We will refer to the two subsets as *region*  $\mathcal{H}$  and *region*  $\mathcal{L}$  and, whenever it proves convenient, to countries belonging to them by  $h \in \mathcal{H}$  and  $l \in \mathcal{L}$ ,

respectively.

Countries in  $\mathcal{H}$  and  $\mathcal{L}$  differ in that they face different random generating processes for their productivity parameters  $\{\eta_{z,v}\}_{z \in \mathbb{Z}}$ . In particular, we assume that, on the one hand, for any  $h \in \mathcal{H}$  and every  $z \in \mathbb{Z}$ , each  $\eta_{z,h}$  is independently drawn from a uniform density function with support  $[\underline{\eta}, \bar{\eta}]$ , where  $\underline{\eta} > 1$ , just like in the previous section. On the other hand, for any  $l \in \mathcal{L}$  and every  $z \in \mathbb{Z}$ , we assume that each  $\eta_{z,l} = \bar{\eta}$ .

Notice that, for countries in region  $\mathcal{H}$ , this setup keeps intact the fact that we let sectoral productivity differentials become increasingly pronounced at higher levels of quality. However, the current setup also allows for the presence of absolute advantages (at the aggregate level) across subsets of countries, which were absent in the model discussed in Section 2. The next proposition characterises equilibrium wages.

**Proposition 3** *Suppose that the set  $\mathbb{V}$  is composed by two disjoint subsets with positive measure:  $\mathcal{H}$  and  $\mathcal{L}$ . Assume that: a) for any  $(z, h) \in \mathbb{Z} \times \mathcal{H}$ ,  $\eta_{z,h}$  is independently drawn uniform density function with support  $[\underline{\eta}, \bar{\eta}]$ ; b) for any  $(z, l) \in \mathbb{Z} \times \mathcal{L}$ ,  $\eta_{z,l} = \bar{\eta}$ . Then:*

- (i) for any  $h \in \mathcal{H}$ ,  $w_h = w_H$ ;
- (ii) for any  $l \in \mathcal{L}$ ,  $w_l = w_L$ ;
- (iii)  $w_H > w_L$ .

**Proof.** In Appendix A. ■

Proposition 3 states that equilibrium wages in region  $\mathcal{H}$  will be higher than in region  $\mathcal{L}$ . The intuition for this result is analogous to all Ricardian models of trade with absolute and comparative advantages. Essentially, region  $\mathcal{H}$  (which displays an absolute advantage over region  $\mathcal{L}$ ) will enjoy higher wages than region  $\mathcal{L}$ , since this is necessary to lower the monetary costs in  $\mathcal{L}$ , and thus allow countries in  $\mathcal{L}$  to export enough to countries in  $\mathcal{H}$  and keep the trade balance in equilibrium. Henceforth, without loss of generality, we take the wage in region  $\mathcal{L}$  as the *numeraire* of the economy, and accordingly set  $w_L = 1$ .

From the results in Proposition 3 it immediately follows that optimal choices will be identical for countries from the same region. That is, for any  $h', h'' \in \mathcal{H}$ , we have that  $\beta_{z,v}^{h'} = \beta_{z,v}^{h''}$  and  $q_{z,v}^{h'} = q_{z,v}^{h''}$ , while for any  $l', l'' \in \mathcal{L}$ , we have that  $\beta_{z,v}^{l'} = \beta_{z,v}^{l''}$  and  $q_{z,v}^{l'} = q_{z,v}^{l''}$ , in both cases for all  $(z, v) \in \mathbb{Z} \times \mathbb{V}$ . In other words, the demand intensity and the consumed quality for a specific variety of a differentiated good is common to all countries belonging to the same region. We thus introduce the following notation, which will be recurrently used in the next subsections:

(i)  $\beta_{z,v}^H$  denotes the demand intensity for  $(z, v) \in \mathbb{Z} \times \mathbb{V}$  by a consumer from region  $\mathcal{H}$ ; (ii)  $\beta_{z,v}^L$  denotes the demand intensity for  $(z, v) \in \mathbb{Z} \times \mathbb{V}$  by a consumer from region  $\mathcal{L}$ .

Recall also that our nonhomothetic preferences imply that the willingness to pay for quality is increasing in the consumer's income. As a consequence, in the presence of cross-country income inequality, consumers from  $\mathcal{H}$  purchase higher quality versions than consumers from  $\mathcal{L}$ . In addition, given the income level, consumers optimally tend to choose a relatively higher quality of consumption for those commodities carrying a relatively lower  $\eta_{z,v}$ . The next proposition formally states these results concerning the consumer choice.

**Proposition 4** *Let  $q_{z,v}^H$  and  $q_{z,v}^L$  denote the quality of consumption of commodity  $(z, v) \in \mathbb{Z} \times \mathbb{V}$  purchased by a consumer from region  $\mathcal{H}$  and from region  $\mathcal{L}$ , respectively. Then, in equilibrium:*

- (i)  $q_{z,v}^H > q_{z,v}^L > 1$ , for all  $(z, v) \in \mathbb{Z} \times \mathbb{V}$ .
- (ii)  $\partial q_{z,h}^H / \partial \eta_{z,h} < 0$  and  $\partial q_{z,h}^L / \partial \eta_{z,h} < 0$ , for all  $(z, v) \in \mathbb{Z} \times \mathcal{H}$ .

*In addition, letting  $i = H, L$ , and denoting by  $q_{z,\bar{\eta}}^i$  (resp.  $q_{z,\underline{\eta}}^i$ ) the value of  $q_{z,h}^i$  corresponding to the commodity  $(z, h) \in \mathbb{Z} \times \mathcal{H}$  such that  $\eta_{z,h} = \bar{\eta}$  (resp.  $\underline{\eta}$ ):*

- (iii)  $q_{z,l}^i = q_L^i$  for all  $(z, l) \in \mathbb{Z} \times \mathcal{L}$ , with  $q_{z,\bar{\eta}}^i < q_L^i < q_{z,\underline{\eta}}^i$ .

**Proof.** In Appendix A. ■

The first result in Proposition 4 is a straightforward implication of our nonhomothetic preferences: richer consumers choose higher quality versions of all consumed goods.

The second result states that, considering all commodities produced within region  $\mathcal{H}$ , the quality of consumption within a given country is a monotonically decreasing function of the labour requirement elasticities of quality upgrading  $\eta_{z,h}$ . In that regard, notice that since all countries in  $\mathcal{H}$  have the same wage, when considering only producers from this region, a larger  $\eta_{z,h}$  will map monotonically into a higher market price.

Finally, the third result shows that, for any given level of consumer income, the quality of the goods produced within region  $\mathcal{L}$  is neither the highest nor the lowest. In particular, the highest quality of each good  $z$  purchased by *any* consumer is produced in the country of region  $\mathcal{H}$  that received the best draw,  $\eta_{z,h} = \underline{\eta}$ . Conversely, the lowest quality of each good  $z$  purchased by *any* consumer is produced in the country of region  $\mathcal{H}$  that received the worst draw,  $\eta_{z,h} = \bar{\eta}$ . In this last case, despite the fact that all producers from  $\mathcal{L}$  received draws equal to  $\bar{\eta}$ , the lower labour cost in  $\mathcal{L}$  allows them to sell higher qualities than the least efficient producers from  $\mathcal{H}$ . Nonetheless, in spite of  $w_H > 1$ , the highest qualities are still provided by the countries with the absolute advantage in the sector.



### 3.1 Export specialisation

We proceed now to study the patterns of exporters' specialisation in this world economy with cross-country inequality. Recall the definition of the RCA from (10). Notice first that the equality of total world demand across all differentiated goods  $z \in \mathbb{Z}$  found in Section 2 still holds true when countries differ in income. As a consequence, in this version of the model, we have again that  $W_z = W$  for all  $z \in \mathbb{Z}$ .

We let  $\lambda \in (0, 1)$  denote the Lebesgue measure of  $\mathcal{H}$ . We can observe that total exports of good  $z$  by country  $v$  are given by

$$X_{z,v} = \lambda \beta_{z,v}^H w_H + (1 - \lambda) \beta_{z,v}^L. \quad (12)$$

Moreover, integrating over  $\mathbb{Z}$ , we obtain the aggregate exports by country  $v$  as

$$X_v = \lambda w_H \int_{\mathbb{Z}} \beta_{z,v}^H dz + (1 - \lambda) \int_{\mathbb{Z}} \beta_{z,v}^L dz. \quad (13)$$

Now, notice that since  $\eta_{z,l} = \bar{\eta}$ , we must have that  $\beta_{z,l}^H = \beta_L^H$  and  $\beta_{z,l}^L = \beta_L^L$ , for all  $(z, l) \in \mathbb{Z} \times \mathcal{L}$ . Hence, denoting by  $RCA_{z,l}$  the revealed comparative advantage of country  $l \in \mathcal{L}$  in good  $z \in \mathbb{Z}$ , using (12) and (13) we obtain:

$$RCA_{z,L} = 1, \quad \text{for all } (z, l) \in \mathbb{Z} \times \mathcal{L}. \quad (14)$$

Consider now a country  $h \in \mathcal{H}$ . Since all  $h$  obtain their draws of  $\eta_{z,h}$  from independent  $U[\underline{\eta}, \bar{\eta}]$  distributions, and since all  $\beta_{z,h}^H$  are well-defined functions of  $\eta_{z,h}$ , applying the law of large numbers it follows that the integrals  $\int_{\mathbb{Z}} \beta_{z,h}^H dz$  and  $\int_{\mathbb{Z}} \beta_{z,h}^L dz$  must both yield an identical value for every country  $h \in \mathcal{H}$ . Let thus denote  $\beta_H^H \equiv \int_{\mathbb{Z}} \beta_{z,h}^H dz$  and  $\beta_H^L \equiv \int_{\mathbb{Z}} \beta_{z,h}^L dz$ , which using (12) and (13) lead to:

$$RCA_{z,h} = \frac{\lambda \beta_{z,h}^H w_H + (1 - \lambda) \beta_{z,h}^L}{\lambda \beta_H^H w_H + (1 - \lambda) \beta_H^L}, \quad \text{for any } (z, h) \in \mathbb{Z} \times \mathcal{H}. \quad (15)$$

Note that the demand intensities  $\beta_{z,h}^i$  are all strictly decreasing functions of the draws  $\eta_{z,h}$ .<sup>13</sup> We can then state the following result, which links again the revealed comparative advantage of an exporter of good  $z$  to the productivity draw of the exporter in that sector.

**Proposition 5** *The revealed comparative advantage of country  $h \in \mathcal{H}$  in good  $z \in \mathbb{Z}$  may be depicted by a strictly decreasing function of the sectoral productivity draw  $\eta_{z,h}$ . Formally,*

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<sup>13</sup>This result is an immediate implication of the second result in Proposition 4.

$RCA_{z,h} = \Psi(\eta_{z,h})$ , where  $\Psi(\eta_{z,h}) : [\underline{\eta}, \bar{\eta}] \rightarrow \mathbb{R}^+$ , with  $\Psi'(\cdot) < 0$ ,  $\Psi(\underline{\eta}) > 1$ , and  $\Psi(\bar{\eta}) < 1$ . Moreover,  $\exists \tilde{\eta} \in (\underline{\eta}, \bar{\eta})$  such that when  $\eta_{z,h} \lesseqgtr \tilde{\eta}$ , then  $RCA_{z,h} \gtrless RCA_{z,L}$ .

**Proof.** In Appendix A. ■

As we did before, by looking at a particular  $z \in \mathbb{Z}$ , we may compare the RCA of different countries in a given sector. We can then observe that Proposition 5 yields an analogous result in terms of export specialisation as Proposition 2: economies with lower  $\eta_{z,h}$  draws tend to display stronger RCA in sector  $z$ . Furthermore, producers from the country that received the best possible draw,  $\eta_{z,h} = \underline{\eta}$ , always display the highest observed value of  $RCA_{z,h}$ . However, in contrast with Proposition 2, in this version of the model the RCA will no longer map monotonically into sectoral absolute advantages. More precisely, since the wage differential between regions  $\mathcal{H}$  and  $\mathcal{L}$  creates a wedge between the absolute and the comparative advantage, it is no longer the case that  $RCA_{z,v}$  can be represented by a monotonically decreasing function of the productivity draw  $\eta_{z,v}$  for all  $v \in \mathbb{V}$ . In fact, although a country  $h \in \mathcal{H}$  with draw  $\tilde{\eta} < \eta_{z,h} < \bar{\eta}$  displays higher labour productivity in sector  $z$  than any country  $l \in \mathcal{L}$ , the RCA in sector  $z$  of country  $h$  is smaller than the one of country  $l$ .

Finally notice that, according to Proposition 4, those producers from  $\mathcal{H}$  that received draws  $\eta_{z,h} = \underline{\eta}$  are also the ones to end up selling the highest qualities of good  $z$  in the world markets. In fact, they sell the highest quality to both consumers from  $\mathcal{H}$  and  $\mathcal{L}$ . As a consequence, merging the results in Proposition 4 and Proposition 5, our model yields an interesting prediction that we will bring to the data in Section 4. Namely, countries that display a stronger revealed comparative advantage in good  $z$  are also those exporting varieties of good  $z$  at higher levels of quality.

### 3.2 Import Specialisation

We turn now to study the implications of this version of the model in terms of import specialisation. For any country  $i$ , the import shares of good  $z$  originating from the country  $v'$  are given by  $\beta_{z,v'}^i / \int_{\mathbb{V}} \beta_{z,v}^i dv$ . However, since the consumer's optimisation problem yields  $\int_{\mathbb{V}} \beta_{z,v}^i dv = 1$  for all  $z \in \mathbb{Z}$ , we can represent those import shares simply by the demand intensities  $\beta_{z,v'}^i$ .

**Proposition 6** Let  $\beta_{z,\underline{\eta}}^H$  and  $\beta_{z,\underline{\eta}}^L$  denote, respectively, the demand intensity by consumers from region  $\mathcal{H}$  and  $\mathcal{L}$  for goods produced by exporters who received a draw  $\eta_{z,h} = \underline{\eta}$ . Then:  $\beta_{z,\underline{\eta}}^H > \beta_{z,\underline{\eta}}^L$ , for all  $z \in \mathbb{Z}$ .

**Proof.** In Appendix A. ■

Proposition 6 states that the import share of any particular good originating from exporters exhibiting the highest RCA in that sector are larger in countries from region  $\mathcal{H}$  than in countries from region  $\mathcal{L}$ . In other words, the share of imports originating from exporters exhibiting the strongest cost advantage in producing a given good should grow with the importer's *per capita* income. This is because the nonhomothetic structure of preferences implies that richer importers tend to buy high-quality commodities, while such commodities are those exhibiting wider cost differentials across countries. To the best of our knowledge, this is a novel prediction in the trade literature that has never been tested empirically. In Section 4.2, we provide evidence consistent with the prediction that richer economies are more likely to buy their imports from producers who display a stronger revealed comparative advantage in the imported goods.

### 3.3 Extension: Cross-country inequality in an $N$ -region world

We consider now an extension to the previous setup where the world is composed by  $K > 2$  regions, indexed by  $k = 1, \dots, K$ . We let  $\mathcal{V}_k \subset \mathbb{V}$  denote the subset of countries from region  $k$ , where  $\mathcal{V}_k$  has Lebesgue measure  $\lambda_k > 0$ . In addition, we let each country in region  $k$  be denoted by a particular  $v_k \in \mathcal{V}_k$ . (The results discussed in this section are formalised in Appendix B.)

Henceforth we assume that for any  $v_k \in \mathcal{V}_k$  and every  $z \in \mathbb{Z}$ , each  $\eta_{z,v_k}$  is independently drawn from the following two-step distribution function:

1. with probability  $(\eta_k - \underline{\eta})/(\bar{\eta} - \underline{\eta})$ , the sectoral productivity is  $\eta_{z,v_k} = \eta_k$ , where  $\eta_k \in [\underline{\eta}, \bar{\eta}]$ , and is described in further detail below;
2. with probability  $(\bar{\eta} - \eta_k)/(\bar{\eta} - \underline{\eta})$ , the value of  $\eta_{z,v_k}$  is drawn from a uniform distribution with support over  $[\eta_k, \bar{\eta}]$ .

We assume that, for any two regions  $k' < k''$ , it holds that  $\eta_{k'} < \eta_{k''}$ , while on the extremes:

$$\eta_k = \begin{cases} \bar{\eta}, & \text{for } k = K, \\ \underline{\eta}, & \text{for } k = 1. \end{cases}$$

(Notice that if we let  $K = 2$ , then this setup would become exactly as the one presented above, with  $\mathcal{H} = 1$  and  $\mathcal{L} = 2$ .)

In this extended setup, equilibrium wages will display an analogous structure as the one described in Proposition 3. Namely, in equilibrium, the wage for all  $v_k \in \mathcal{V}_k$  will be  $w_k$ . In addition, equilibrium wages are such that  $w_1 > \dots > w_{k'} > \dots > w_K$ , where  $1 < k' < K$ .

We now use the superindex  $j = 1, 2, \dots, K$  to denote the region of origin of the consumer. (Notice that, since all individuals from the same region earn the same wages, they must choose identical consumption profiles.) We then let  $\beta_{z,v_k}^j$  denote the demand intensity by a consumer from region  $\mathcal{V}_j$  for good  $(z, v_k) \in \mathbb{Z} \times \mathcal{V}_k$ . It follows then that, for a country  $v_k$

$$X_{z,v_k} = \sum_{j=1}^K \lambda_j w_j \beta_{z,v_k}^j.$$

Moreover, in equilibrium, it must be the case that  $X_{v_k} = w_k$  for all  $v_k \in \mathcal{V}_k$ . In addition,  $W_z = W$  for all  $z \in \mathbb{Z}$  is still true in this extended setup. As a result, the RCA of country  $v_k$  in good  $z$  is given by

$$RCA_{z,v_k} = \frac{\sum_{j=1}^K \lambda_j w_j \beta_{z,v_k}^j}{w_k}. \quad (16)$$

Since wages differ across regions, once again, we cannot find a monotonic relationship between  $RCA_{z,v_k}$  in (16) and the productivity draws  $\eta_{z,v_k}$  of all countries in the world pooled together. However, we can still find a result analogous to Proposition 5. In particular, it is still true that the highest value of  $RCA_{z,v_k}$  corresponds to the country in region  $\mathcal{V}_1$  receiving the best possible draw in sector  $z$ . That is,  $RCA_{z,v_k}$  is highest for country  $v_1$  with  $\eta_{z,v_1} = \underline{\eta}$ .

Lastly, concerning imports specialisation, this extension also yields a result that is analogous to that in Proposition 6. For the same reasons as discusses in Section 3.2, the demand intensities  $\beta_{z,v_k}^j$  equal the import shares in country  $j$  of good  $z$  originating from exporter  $v_k$ . Now, following an analogous notation as in Proposition 6, we can show that  $\beta_{z,\underline{\eta}}^1 > \dots > \beta_{z,\underline{\eta}}^{k'} > \dots > \beta_{z,\underline{\eta}}^K$ , where  $1 < k' < K$ . Again, this result stems from the fact that our preferences are nonhomothetic in quality, hence richer consumers allocate a larger share of their spending in good  $z$  to the producers who can most efficiently offer higher qualities versions of that good.

## 4 Empirical analysis

In this section we bring some of the main results of our theoretical model to the data. We divide the section in two parts. The first part presents evidence consistent with the notion that export specialisation at the product level becomes greater at higher levels of quality of production. The second deals with our model's prediction regarding import specialisation at different income levels. In particular, it provides evidence consistent with the hypothesis that richer countries import relatively more from exporters displaying stronger comparative advantage in the goods being imported.

## 4.1 Exporters behaviour

Our theory is fundamentally based on the assumption that sectoral productivity differentials across countries become wider along their respective quality ladders. In its purest sense, this assumption is really hard to test empirically. However, the intensification of sectoral productivity differentials at higher qualities implies that the degree of specialisation of countries in specific goods and the level of quality of their exports should display a positive correlation. In this subsection we aim to provide some evidence consistent with this prediction.

Objective data on products quality is hardly available for a large set of goods.<sup>14</sup> For that reason, we take unit values as a proxy for the quality of the commodity.<sup>15</sup> Like in the previous sections, in order to measure the degree of specialisation we use the revealed comparative advantage (RCA). That is, for each exporter  $x$  of good  $z$  in year  $t$ , we compute the ratio:

$$RCA_{z,x,t} \equiv \frac{(V_{z,x,t}/V_{x,t})}{(W_{z,t}/W_t)};$$

where  $V_{z,x,t}$  (resp.  $W_{z,t}$ ) is the total value of exports of good  $z$  by country  $x$  (resp. by the world) in year  $t$ , and  $V_{x,t}$  (resp.  $W_t$ ) is the aggregate value of exports by country  $x$  (resp. by the world) in year  $t$ .

We compute unit values of exports using the dataset compiled by Gaulier and Zignano (2010). This database reports monetary values and physical quantities of bilateral trade for years 1995 to 2009 for more than 5000 products categorised according to the 6-digit Harmonised System (HS-6). Monetary values are measured FOB (free on board) in US dollars. We use the same dataset to compute the RCA of each exporter in each particular HS-6 product.

In our model, comparative advantages become stronger at higher levels of quality of production. Taking unit values as proxy for quality, this implies that the *average* unit values of exports by each country in each of the traded goods should correlate positively with the RCA of the exporter in those goods.

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<sup>14</sup>The only article we are aware of assessing the effects of product quality on export performance using objective measures of quality is Crozet, Head and Mayer (2011) for the champagne industry in France.

<sup>15</sup>There is a large literature in trade using unit values as proxy for quality: e.g., Schott (2004), Hallak (2006), Fieler (2011a). We acknowledge the fact that unit values are not perfect proxies for quality, since other factors may also affect prices, such as: the degree of horizontal differentiation across industries, heterogeneous transport costs, trade tariffs. In addition, as shown by Simonovska (2011), nonhomothetic preferences may induce firms to charge variable mark-ups on their products depending on the income level of the importer. See Khandelwal (2010) and Hallak and Schott (2011) for some innovative methods to infer quality from prices taking into account both horizontal and vertical differentiation of products.

To assess this implication, we first run the following regression:

$$\log(\textit{weighted\_mean\_}P_{z,x,t}) = \alpha + \beta \log(RCA_{z,x,t}) + \delta_z + \zeta_t + \nu_{z,x,t}. \quad (17)$$

The dependent variable in (17) is the logarithm of the average unit value of exports across importers, using export shares as weights for each importer's unit value.<sup>16</sup> The regression also includes product dummies  $\delta_z$  (to control for different average prices of goods across different categories of the HS-6 system) and time dummies  $\zeta_t$  (to control for aggregate price levels, which may well differ across years). The results of regression (17) are shown in column (1) of Table 1.A. Consistent with our model, the variables  $\log(RCA_{z,x,t})$  and  $\log(\textit{weighted\_mean\_}P_{z,x,t})$  display a positive correlation, which is also highly significant.<sup>17</sup>

It might be the case that the above correlation is simply reflecting the fact that more developed economies tend to capture larger markets for their products and, at the same time, tend to produce higher quality versions of the traded products. To account for that possibility, in column (2) we include the logarithm of exporter's income per capita. As we can observe, the coefficient associated to this variable is indeed positive and highly significant.<sup>18</sup> Nevertheless, our estimate of  $\beta$  remains essentially unaltered and highly significant, suggesting that the correlation between RCA and export unit values is not solely driven by differences in the exporters' income per head.

In column (3) we add a set of exporter dummies to the regression. The rationale for this is to control for fixed (or slow-changing) exporters' characteristics (such as, geographic location, institutions, openness to trade) which may somehow affect average export prices, and may be at the same time correlated with export penetration. Our correlation of interest falls a bit in magnitude, but still remains positive and highly significant. Finally, in column (4) we include a

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<sup>16</sup>More precisely, the dependent variable is computed as follows:

$$\log(\textit{weighted\_mean\_}P_{z,x,t}) \equiv \log\left(\sum_{m \in M} \frac{v_{z,x,m,t}}{v_{z,x,t}} \times \frac{v_{z,x,m,t}}{c_{i,x,m,t}}\right);$$

where:  $v_{z,x,m,t}$  (resp.  $c_{z,x,m,t}$ ) denotes the monetary value (resp. the physical quantity) of exports of good  $z$ , by exporter  $x$ , to importer  $m$ , in year  $t$ . The summation is across the set of importers,  $M$ . To mitigate the possible contaminating effects of outliers, we have discarded unit values above the 95th percentile and below the 5th percentile for each exporter and product (our results remain essentially intact if we do not trim the price data at the two extremes of the distribution).

<sup>17</sup>A similar regression is run by Alcala (2011), although for a smaller set of goods (he uses only the apparel industry) and only using import prices by the US as the dependent variable. The results he obtains are very similar to ours in Table 1.A.

<sup>18</sup>This result is consistent with the previous evidence in the literature: *e.g.*, Schott (2004) and Hallak (2006).

**TABLE 1.A**

	Dependent Variable: log of (weighted) mean unit value of exports			
	(1)	(2)	(3)	(4)
Log RCA	0.039*** (0.008)	0.045*** (0.009)	0.037*** (0.007)	0.068*** (0.006)
Log GDP per capita exporter		0.319*** (0.041)	0.366* (0.207)	0.441** (0.213)
Year dummies	YES	YES	YES	YES
Product dummies	YES	YES	YES	-
Exporter dummies	NO	NO	YES	-
Product-Exporter dummies	-	-	-	YES
Observations	4,405,953	4,176,504	4,176,504	4,176,504
Adj. R squared	0.66	0.68	0.72	0.81

Robust absolute standard errors clustered at the exporter level reported in parentheses. All data is for years 1995-2009.

The total number of different products is 5017. \* significant at 10%; \*\* significant at 5%; \*\*\* significant at 1%.

full set of product-exporter fixed effects. These dummies would control for fixed characteristics of exporters in specific markets: for example, geographic distance from the exporter to the main importers of a given product, the intensity of competition in given industries across different exporters, etc. After including product-exporter dummies, our estimate of the correlation between log of RCA and log of export unit values remains positive and highly significant, rising also in magnitude by a fair amount. In addition, the estimate associated to the exporter's income per head also remains positive and significant.

Table 1.A shows pooled regressions for all HS-6 products. However, the correlation of interest may well differ across industries. To get a feeling of whether the previous results are mainly driven by particular sectors, we next split the set of HS 6-digit products according to fourteen separate subgroups at the 2-digit level.<sup>19</sup> In Table 1.B, we repeat the regression conducted in column (4), but running it separately for each of the 14 subgroups. Although the point estimates for  $\beta$  tend to differ across subgroups, in all cases they come out positive and highly significant (except for 'Mineral Products' where it is actually negative and significant). Interestingly (and quite expectably), the point estimates for  $\beta$  and for the correlation with the exporter's income per capita are largest for 'Machinery/Electrical' and 'Transportation' products, which comprise manufacturing industries producing highly differentiated products in terms of intrinsic quality.

<sup>19</sup>The subgroups in Table 1.B are formed by merging together subgroups at 2-digit aggregation level, according to <http://www.foreign-trade.com/reference/hscodet.htm>. We excluded all products within the subgroups 'Miscellaneous' and 'Service'.

**TABLE 1.B**

	animal & anim. prod.	vegetable products	foodstuff	mineral products	chem. & allied ind.	plastic & rubbers	skin, leath. & furs
log RCA	0.063*** (0.006)	0.028*** (0.007)	0.039*** (0.008)	-0.021** (0.009)	0.052*** (0.010)	0.034*** (0.008)	0.078*** (0.007)
log Ypc exporter	0.354** (0.140)	0.271* (0.171)	0.372** (0.188)	0.298 (0.213)	0.238 (0.411)	0.398** (0.194)	0.479*** (0.140)
Observations	131,841	243,517	172,096	91,124	483,160	186,173	62,602
# of products	194	323	181	151	760	189	74
Adj. R squared	0.68	0.69	0.68	0.59	0.76	0.64	0.76

  

	wood & wood prod.	textiles	footwear	stone & glass	metals	machinery & electrical	transport.
log RCA	0.027*** (0.008)	0.047*** (0.008)	0.079*** (0.009)	0.073*** (0.010)	0.028*** (0.008)	0.128*** (0.006)	0.160*** (0.007)
log Ypc exporter	0.298* (0.168)	0.324 (0.269)	0.441* (0.243)	0.185 (0.290)	0.469** (0.214)	0.702*** (0.183)	0.657*** (0.150)
Observations	206,601	695,506	54,675	165,753	459,481	743,870	121,006
# of products	228	809	55	188	587	762	132
Adj. R squared	0.73	0.70	0.65	0.91	0.75	0.65	0.66

Robust absolute standard errors clustered at the exporter level reported in parentheses. All data is for years 1995-2009.

All regression include time dummies and product-exporter dummies. \* significant 10%; \*\* significant 5%; \*\*\* significant 1%.

## 4.2 Importers behaviour

Another key aspect of our theory is how imports respond to variations in incomes. The model predicts that changes in incomes will lead to: (i) changes in the quality of consumption, and (ii) changes in the distribution of total production across different economies. The former result stems from our nonhomothetic preferences, while the latter derives from the interaction between nonhomotheticity and the increasing heterogeneity of sectoral productivities at higher levels of quality.

Concerning the first prediction, there is vast evidence showing that richer consumers buy their imports in higher quality levels than poorer consumers do: *e.g.*, Hallak (2006, 2010), Choi et al. (2009), Fieler (2011a). In particular, using unit values to proxy for product quality, Fieler (2011a) shows that import prices correlate positively with the level of income per head of the importer, even when looking at products originating from the same exporter and HS-6 category.

The previous literature linking import prices and the importer's GDP per head has then provided evidence consistent with the hypothesis that richer individuals purchase goods in higher quality levels. However, that literature has mostly remained silent as to where those imports tend to originate from. In that regard, our model also yields an interesting prediction regarding imports specialisation: if it is true that taste for quality rises with income and that countries' comparative advantage in production become more pronounced at higher levels of quality, then



richer countries should purchase a larger share of their imports of given goods from economies displaying a comparative advantage in those goods.

In what follows we aim at providing evidence of such relationship between importer's income per head and origin of imports. (For computational purposes, given the large number of observations in the panel, in Table 2.A, we use only data from 2009, which is the last year available in the dataset.)<sup>20</sup>

In Table 2.A, we regress the share of imports of good  $z$  by importer  $m$  originating from exporter  $x$  on the RCA of  $x$  in  $z$  interacted with the importer's income per head ( $Y_m$ ). More precisely, we conduct the following regression (where  $impo_{z,m,x}$  denotes the value of imports of good  $z$  by importer  $m$  originating from exporter  $x$ , and  $X$  denotes the set of exporters in the sample):

$$\log \left( \frac{impo_{z,m,x}}{\sum_{x \in X} impo_{z,m,x}} \right) = \rho \log(RCA_{z,x}) + \theta [\log(Y_m) \times \log(RCA_{z,x})] + \mathbf{G}_{m,x} + \delta_z + \mu_m + \varepsilon_x + \nu_{z,x,m}. \quad (18)$$

Our model predicts a positive value for  $\theta$ . This would suggest that richer importers tend to buy a larger share of the imports of good  $z$  from exporters exhibiting a comparative advantage in  $z$ . Regression (18) includes product dummies ( $\delta_z$ ), importer dummies ( $\mu_m$ ), exporter dummies ( $\varepsilon_x$ ), and a set of bilateral gravity terms ( $\mathbf{G}_{m,x}$ ) taken from Mayer and Zignano (2006).

Before strictly running regression (18), in column (1) of Table 2.A, we first regress the dependent variable of (18) against *only* the log of the RCA of exporter  $x$  in good  $z$  (together with product, importer and exporter dummies), which shows as we would expect that those two variables are positively correlated. Secondly, in column (2), we report the results of the regression that includes the interaction term. We can see that the estimated  $\theta$  is positive and highly significant, consistent with our theory. Finally, in column (3), we add six traditional gravity terms, and we can observe the previous results remain essentially intact. We can also observe that the estimates for each of the gravity terms are significant, and they all carry the expected sign.

One possible concern with regression (18) is the fact that  $RCA_{z,x}$  is computed with the *same* data that is used to construct the dependent variable. In terms of our estimation of  $\theta$ , this could represent an issue if a very large economy turns out to be also a very rich economy (for example,

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<sup>20</sup>As robustness checks, we have also run the regressions reported in Table 2.A separately for all the years in the sample. All the results for years 1995-2008 are qualitatively identical, and very similar in magnitude, to those of year 2009. These additional results are available from the authors upon request.

the US). In that case, since the imports of good  $z$  by such large and rich economy will be strongly influencing the independent variable  $RCA_{z,x}$ , we may be somehow generating by construction a positive correlation between the dependent variable and  $[\log(Y_m) \times \log(RCA_{z,x})]$ .

In order to deal with this concern, in column (4) we split the set of 184 importers in two separate subsets of 92 importers each (subset  $A$  and subset  $B$ ). When splitting the original set of 184 importers, we do so in such a way the two subsets display similar GDP per capita distributions. (See Appendix C for details and descriptive statistics of the two sub-samples.) We next use the subset  $A$  to compute the revealed comparative advantage of each exporter in each product ( $RCA_{z,x}$ ), while we use the subset  $B$  for the dependent variable in the regression. By construction, there is therefore no link between the dependent variable in (18) and  $RCA_{z,x}$ , since those two variable are computed with data from different sets of importers.

As we may readily observe, the results in column (4) of Table 2.A confirm our previous results in column (3) – the estimate for  $\theta$  is positive and highly significant, and of very similar magnitude as in column (3). Lastly, in column (5) we use the RCA computed with the subset  $A$  of importers to instrument the RCA used in column (3); again the obtained results confirm our previous findings.<sup>21</sup>

The regressions in Table 2.A pool together more than 5000 6-digit products, implicitly assuming the same coefficients for all of them. This might actually be a strong assumption to make. In Table 2.B we divide again the 6-digit products into 14 subgroups (the same subgroups we used before in Table 1.B). In the sake of brevity, we report only the estimates for  $\rho$  and  $\theta$  in (18). As we can observe, the estimates for each subgroup follow a similar pattern as those in Table 2.A; in particular, the estimate associated to the interaction term is always positive and highly significant for each subgroup. As further robustness check, in Table 2.C, we report the percentage of positive and negative estimates for  $\theta$  when we run a separate regression for each of the products in the HS 6-digit categorisation.

To conclude, taken jointly, Section 4 yields support to the following ideas: (i) as getting richer, countries tend to raise the quality of the goods they consume (positive correlation between import prices and income per head of importer previously found in the literature); (ii) this, in turn, leads them to raise their import shares originating from exporters displaying a comparative

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<sup>21</sup>See Table 2.A (extended) in Appendix C, for some additional robustness checks. There, in column (2) and (5), we control for product-importer fixed effects ( $\varsigma_{z,m}$ ), instead of  $\delta_z$  and  $\mu_m$  separately as in (18). In addition, in columns (3) and (4) we exclude high income countries from the OECD and high income countries as classified by the World Bank, to see whether the previous results are mainly driven by the behaviour of richer economies. As it may be readily observed, our correlation of interest,  $\theta$  in (18), remains always positive and highly significant.

**Table 2.A**

	Dependent Variable: log impo shares of product $i$ from exporter $x$				
	Full Sample			Restricted Sample	
	(1)	(2)	(3)	(4)	(5) - 2SLS
Log RCA exporter	0.456***	-0.676***	-0.469***	-0.422***	-0.594***
	-0.026	(0.138)	(0.106)	(0.092)	(0.129)
Interaction term		0.119***	0.104***	0.088***	0.125***
		(0.015)	(0.012)	(0.010)	(0.014)
Distance expo-impo ( $\times 1000$ )			-0.121***	-0.116***	-0.121***
			(0.009)	(0.010)	(0.010)
Contiguity			1.098***	1.116***	1.162***
			(0.101)	(0.131)	(0.132)
Common official language			0.362***	0.413***	0.436***
			(0.099)	(0.133)	(0.133)
Common coloniser			0.255*	0.164	0.219
			(0.152)	(0.178)	(0.179)
Common legal origin			0.204***	0.204**	0.222**
			(0.082)	(0.096)	(0.096)
Common currency			0.351**	0.415**	0.408**
			(0.149)	(0.174)	(0.174)
Observations	5,773,873	5,773,873	5,571,567	2,709,459	2,709,459
Number of importers	184	184	184	92	92
R squared	0.47	0.47	0.53	0.51	0.51

Robust absolute standard errors clustered at the importer and exporter level reported in parentheses. All data corresponds to the year 2009.

All regressions include product dummies, importer dummies and exporter dummies. The total number of HS 6-digit products is 5017.

Column (4) uses importers in *subset A* to compute the exporters' RCA and importers in *subset B* to compute the dependent variable. Column (5) uses the RCA computed with importers in *subset A* to instrument the exporters' RCA. \* significant 10%; \*\* significant 5%; \*\*\* significant 1%.

**Table 2.B**

	animal & anim. prod.	vegetable products	foodstuff	mineral products	chem. & allied ind.	plastic & rubbers	skin, leath. & furs
log RCA	-0.322***	-0.298***	-0.344***	-0.269**	-0.500***	-0.548***	-0.622***
	(0.106)	(0.104)	(0.096)	(0.145)	(0.138)	(0.138)	(0.155)
interaction term	0.073***	0.079***	0.089***	0.075***	0.107***	0.118***	0.120***
	(0.012)	(0.011)	(0.011)	(0.015)	(0.015)	(0.015)	(0.016)
Observations	105,332	210,866	215,975	72,839	602,592	317,328	66,347
Adj. R squared	0.44	0.49	0.50	0.46	0.49	0.52	0.60

  

	wood & wood prod.	textiles	footwear	stone & glass	metals	machinery & electrical	transport.
log RCA	-0.444***	-0.411***	-0.644***	-0.527***	-0.541***	-0.711***	-0.554***
	(0.105)	(0.166)	(0.155)	(0.131)	(0.130)	(0.131)	(0.112)
interaction term	0.101***	0.090***	0.119***	0.107***	0.111***	0.134***	0.114***
	(0.012)	(0.019)	(0.016)	(0.015)	(0.015)	(0.014)	(0.013)
Observations	252,135	795,926	75,522	209,397	630,910	1,296,090	176,916
Adj. R squared	0.53	0.55	0.61	0.53	0.50	0.55	0.53

Robust absolute standard errors clustered at the importer-exporter level in parentheses. All data corresponds to year 2009.

All regression include product, exporter and importer dummies, and the set of gravity terms used before in Table 2.A taken from Mayer & Zignano (2006). \* significant 10%; \*\* significant 5%; \*\*\* significant 1%.

**Table 2.C**

<b>Coefficients of Log(Yn) x Log(RCA): independent regressions for each HS 6-digit product</b>						
% positive coefficients			% negative coefficients			median coefficient
insignificant	significant 10%	significant 1%	insignificant	significant 10%	significant 1%	
29.8%	15.7%	38.0%	14.3%	1.6%	0.5%	0.076
	83.5%			16.4%		

Total number of different products was 4904 (98 products were lost due to insufficient observations). Data corresponds to year 2009  
Regressions include importer dummies and the set of gravity terms used in Table 2.A taken from Mayer & Zignano (2006).

advantage in those goods; (iii) this alteration in the origin of imports would reflect the fact that these are the exporters relatively more productive at providing higher quality varieties of those goods.

## 5 Conclusion

We presented a Ricardian model of trade with the distinctive feature that comparative advantages reveal themselves gradually over the course of development. The key factors behind this process are the individuals' continuing upgrading in quality of consumption combined with productivity differentials that widen up as countries seek to increase the quality of their production. As incomes grow and wealthier consumers raise the quality of their consumption baskets, cost differentials between countries become more pronounced. The emergence of such heterogeneities, in turn, alters trade flows, as each economy gradually specialises in producing the subset of goods for which they enjoy a rising comparative advantage.

Our model yielded a number of implications that find empirical support. In this respect, using bilateral trade data at the product level, we showed that the share of imports originating from exporters more intensely specialised in a given product correlates positively with GDP per head of the importer. This is consistent with the model's prediction that richer consumers tend to buy a larger share of their consumption of specific goods from countries exhibiting a comparative advantage in those goods. We also provided some evidence supporting the central assumption of our model, namely the intensification of comparative advantage at higher quality levels. In particular, we found that the degree of export specialisation of countries in specific goods and the level of quality of their exports display a positive correlation. This fact is consistent with the idea that Ricardian specialisation tends to become more intense at the upper levels of quality of production.

As a last remark, our model has assumed away any sort of trade frictions. In a sense, this

was a deliberate choice, so as to illustrate our proposed mechanism as cleanly as possible. Yet, incorporating trade costs could actually represent a promising extension to the core model. In this respect, owing to the widening of productivity differentials at higher quality of production, a natural implication of the model would be that trade costs will generate milder distortions on trade flows as the quality of production rises. This implication could help rationalizing some empirical observations found in the trade literature, such as the positive relationship between the imports/GDP ratio and the importer's GDP per head.

## Appendix A: Omitted proofs

**Solution of Problem (6).** Let  $\mu^i$  denote the Lagrange multiplier associated to the budget constraint, and by  $\delta_{z,v}^i$  the Lagrange multipliers associated to each constraint  $q_{z,v} \geq 1$ . Then, optimisation requires the following FOCs:

$$\ln \beta_{z,v}^i - \eta_{z,v} \ln q_{z,v}^i + \ln(1 + \kappa) + \ln\left(\frac{w_i}{w_v}\right) + \delta_{z,v}^i = 0, \text{ for all } (z, v) \in \mathbb{Z} \times \mathbb{V} \quad (19)$$

$$\frac{q_{z,v}^i}{\beta_{z,v}^i} - \mu^i = 0, \text{ for all } (z, v) \in \mathbb{Z} \times \mathbb{V} \quad (20)$$

$$q_{z,v}^i - 1 \geq 0, \quad \delta_{z,v}^i \geq 0, \quad \text{and} \quad (q_{z,v}^i - 1) \delta_{z,v}^i = 0, \text{ for all } (z, v) \in \mathbb{Z} \times \mathbb{V} \quad (21)$$

$$\int_{\mathbb{Z}} \int_{\mathbb{V}} \beta_{z,v}^i dv dz = 1 \quad (22)$$

Notice first that each (20) may be re-written as  $q_{z,v}^i = \mu^i \beta_{z,v}^i$ . Hence, integrating both sides of the equation over  $\mathbb{Z}$  and  $\mathbb{V}$ , and making use of (22), we may obtain:

$$\int_{\mathbb{Z}} \int_{\mathbb{V}} q_{z,v}^i dv dz = \mu^i; \quad (23)$$

which in turn implies that

$$\beta_{z,v}^i = \frac{q_{z,v}^i}{\mu^i}. \quad (24)$$

Notice also that  $\mu^i \geq 1$ , since  $q_{z,v}^i \geq 1$  and both  $\mathbb{Z}$  and  $\mathbb{V}$  have unit mass. ■

**Lemma 2** *If  $w_v = w$  for all  $v \in \mathbb{V}$ , then  $\delta_{z,v}^i = 0$  for all  $(z, v) \in \mathbb{Z} \times \mathbb{V}$  and all  $i \in \mathbb{V}$ .*

**Proof.** Replacing (24) into (19), and noting that  $w_i = w$  when  $w_v = w$  for all  $v \in \mathbb{V}$ , we obtain:

$$\ln(1 + \kappa) + \delta_{z,v}^i = (\eta_{z,v} - 1) \ln q_{z,v}^i + \ln \mu^i, \text{ for all } (z, v) \in \mathbb{Z} \times \mathbb{V}. \quad (25)$$

Now, suppose that there exist  $(z', v')$  and  $(z'', v'')$  such that  $\delta_{z',v'}^i > \delta_{z'',v''}^i = 0$ . Then, from (21) and (25) it follows that  $\delta_{z',v'}^i = \ln \mu^i - \ln(1 + \kappa) > (\eta_{z'',v''} - 1) \ln q_{z'',v''}^i + \ln \mu^i - \ln(1 + \kappa)$ , but this is impossible since  $\eta_{z'',v''} > 1$  and  $q_{z'',v''}^i \geq 1$ .

Alternatively, suppose that  $\delta_{z,v}^i > 0$  for all  $(z, v) \in \mathbb{Z} \times \mathbb{V}$ . Then, because of (21) we have that  $q_{z,v}^i = 1$  for all  $(z, v) \in \mathbb{Z} \times \mathbb{V}$ , which in turn leads to  $\mu^i = 1$  owing to (23). However, this means that  $\ln(1 + \kappa) + \delta_{z,v}^i = 0$ , which is impossible when  $\delta_{z,v}^i > 0$  because  $\kappa > 0$ . As a result, it must be the case that when  $w_v = w$  for all  $v \in \mathbb{V}$ , then  $\delta_{z,v}^i = 0$  holds for all  $(z, v) \in \mathbb{Z} \times \mathbb{V}$  and all  $i \in \mathbb{V}$ . ■

**Proof of Lemma 1.** Firstly, note that Lemma 2 (see lemma and proof above) implies that  $\delta_{z,v}^i = 0$  for all  $(z, v) \in \mathbb{Z} \times \mathbb{V}$  and all  $i \in \mathbb{V}$ . Hence, using (23), (24) and (25), we may obtain (7) and (8). Secondly, note that, when  $w_v = w$  for all  $v \in \mathbb{V}$ , using again (25) leads to  $\ln(1 + \kappa) - \ln \mu^i = (\eta_{z,v} - 1) \ln q_{z,v}^i$  for all  $(z, v) \in \mathbb{Z} \times \mathbb{V}$ . Defining now  $\Upsilon^i(\kappa) \equiv \ln(1 + \kappa) - \ln \mu^i$ , we can observe that:

$$\frac{\partial \Upsilon^i}{\partial \kappa} = \frac{(\eta_{z,v} - 1)}{q_{z,v}^i} \frac{\partial q_{z,v}^i}{\partial \kappa}. \quad (26)$$

But, given that  $(\eta_{z,v} - 1) > 0$ , then all  $\partial q_{z,v}^i / \partial \kappa$  must necessarily carry the same sign. Suppose then that  $\partial q_{z,v}^i / \partial \kappa \leq 0$ , for all  $(z, v) \in \mathbb{Z} \times \mathbb{V}$ . Recalling (23), it follows that  $\partial \mu^i / \partial \kappa \leq 0$  as well. But, since  $\partial \Upsilon^i / \partial \kappa = (1 + \kappa)^{-1} - (\mu^i)^{-1} \partial \mu^i / \partial \kappa$ , the fact that  $\partial \mu^i / \partial \kappa \leq 0$  implies that  $\partial \Upsilon^i / \partial \kappa > 0$ , which in turn contradicts the fact that  $\partial q_{z,v}^i / \partial \kappa \leq 0$  for all  $(z, v) \in \mathbb{Z} \times \mathbb{V}$ . As a result, it must be the case that  $\partial q_{z,v}^i / \partial \kappa > 0$  for all  $(z, v) \in \mathbb{Z} \times \mathbb{V}$ . ■

**Proof of Proposition 1.** As a first step, we proceed to prove that  $w_v = w$  for all  $v \in \mathbb{V}$  is an equilibrium of the model. Firstly, notice that when  $w_i = w$  for all  $i \in \mathbb{V}$ , the Lagrange multipliers will be identical for all countries, and in particular we may write  $\mu^i = \mu$  for all  $i \in \mathbb{V}$ . Secondly, using Lemma 1, when  $w_v = w$  for all  $v \in \mathbb{V}$ , conditions in (19) together with (24) and  $\mu^i = \mu$  for all  $i \in \mathbb{V}$ , lead to:

$$q_{z,v}^i = q_{z,v} = \left( \frac{1 + \kappa}{\mu} \right)^{1/(\eta_{z,v} - 1)}, \quad (27)$$

$$\beta_{z,v}^i = \beta_{z,v} = \mu^{-\eta_{z,v}/(\eta_{z,v} - 1)} (1 + \kappa)^{1/(\eta_{z,v} - 1)}. \quad (28)$$

Now, recall that each  $\eta_{z,v}$  is drawn from an independent uniform probability distribution with support  $[\underline{\eta}, \bar{\eta}]$ . Hence, by the law of large numbers, for each country  $v \in \mathbb{V}$ , the (infinite) sequence of draws  $\{\eta_{z,v}\}_{z \in \mathbb{Z}}$  will also be uniformly distributed over  $[\underline{\eta}, \bar{\eta}]$  along the goods space. This implies that, integrating over  $\mathbb{Z}$  and bearing in mind (28),  $\int_{\mathbb{Z}} \beta_{z,v}^i dz = \int_{\mathbb{Z}} \beta_{z,v} dz = \beta_v = \beta > 0$ , for each good  $v \in \mathbb{V}$ . Next, replacing  $\int_{\mathbb{Z}} \beta_{z,v}^i dz = \beta$  into (22), and swapping the order of integration, we obtain  $\int_{\mathbb{V}} \beta dv = 1$ , which in turn implies that  $\beta = 1$  since  $\mathbb{V}$  has unit mass. Then, it is easy to check that all conditions (9) hold simultaneously when  $w_v = w$  for all  $v \in \mathbb{V}$ . We now proceed to prove the above equilibrium is unique. Normalise  $w = 1$ , and suppose for a subset  $\mathcal{J} \subset \mathbb{V}$  of countries with measure  $\lambda_j > 0$  we have  $w_j > 1$ , while for a (disjoint) subset  $\mathcal{K} \subset \mathbb{V}$  of countries with measure  $\lambda_k > 0$  we have  $w_k < 1$ . Denote finally by  $\mathcal{I} \subset \mathbb{V}$  the (complementary) subset of countries with  $w_i = 1$ . Consider some  $k \in \mathcal{K}$ ,  $i \in \mathcal{I}$ , and  $j \in \mathcal{J}$ , and take  $(z_k, k), (z_i, i), (z_j, j)$  such that:  $\eta_{z_k, k} = \eta_{z_i, i} = \eta_{z_j, j} = \eta$ . Notice that, due to the law of large numbers, for any  $\eta \in [\underline{\eta}, \bar{\eta}]$  the measure of good-variety couples for which the last condition is

satisfied is the same in  $k$ ,  $i$  and  $j$ .

As a first step, take country  $i \in \mathcal{I}$ . (19) and (20) imply that, for  $(z_k, k)$ ,  $(z_i, i)$  and  $(z_j, j)$ , we must have, respectively:

$$\begin{aligned}\ln(1 + \kappa) &= \eta \ln(\mu^i) + \ln(w_k) + (\eta - 1) \ln(\beta_{z_k, k}^i) - \delta_{z, k}^i \\ &= \eta \ln(\mu^i) + (\eta - 1) \ln(\beta_{z_i, i}^i) - \delta_{z, i}^i \\ &= \eta \ln(\mu^i) + \ln(w_j) + (\eta - 1) \ln(\beta_{z_j, j}^i) - \delta_{z, j}^i.\end{aligned}$$

Notice also from (21) and (24) that if  $\delta_{z, v}^i > 0$ , then  $\ln \beta_{z, v}^i = -\ln \mu^i$ , whereas if  $\delta_{z, v}^i = 0$ , then  $\ln \beta_{z, v}^i \geq -\ln \mu^i$ . Then,  $\beta_{z_k, k}^i \geq \beta_{z_i, i}^i \geq \beta_{z_j, j}^i$ .

As a second step, take country  $k \in \mathcal{K}$ . (19) and (20) imply that, for  $(z_k, k)$ ,  $(z_i, i)$  and  $(z_j, j)$ , we must have, respectively:

$$\begin{aligned}\ln(1 + \kappa) &= \eta \ln(\mu^k) + (\eta - 1) \ln(\beta_{z_k, k}^k) - \delta_{z, k}^k \\ &= \eta \ln(\mu^k) + \ln\left(\frac{1}{w_k}\right) + (\eta - 1) \ln(\beta_{z_i, i}^k) - \delta_{z, i}^k \\ &= \eta \ln(\mu^k) + \ln\left(\frac{w_j}{w_k}\right) + (\eta - 1) \ln(\beta_{z_j, j}^k) - \delta_{z, j}^k.\end{aligned}$$

Following an analogous reasoning as before, it follows that  $\beta_{z_k, k}^k \geq \beta_{z_i, i}^k \geq \beta_{z_j, j}^k$ .

As a third step, take country  $j \in \mathcal{J}$ , and notice  $w_j > 1$ . (19) and (20) imply that, for  $(z_k, k)$ ,  $(z_i, i)$  and  $(z_j, j)$ , we must have, respectively:

$$\begin{aligned}\ln(1 + \kappa) &= \eta \ln(\mu^j) + \ln\left(\frac{w_k}{w_j}\right) + (\eta - 1) \ln(\beta_{z_k, k}^j) - \delta_{z, k}^j \\ &= \eta \ln(\mu^j) + \ln\left(\frac{1}{w_j}\right) + (\eta - 1) \ln(\beta_{z_i, i}^j) - \delta_{z, i}^j \\ &= \eta \ln(\mu^j) + (\eta - 1) \ln(\beta_{z_j, j}^j) - \delta_{z, j}^j.\end{aligned}$$

Again, with an analogous reasoning as in the previous cases, it then follows that  $\beta_{z_k, k}^j \geq \beta_{z_i, i}^j \geq \beta_{z_j, j}^j$ .

Finally, integrate among the good space  $\mathbb{Z}$  and country space  $\mathbb{V}$ . The above results lead to:

$$\begin{aligned}\lambda^j w_j \int_{\mathbb{Z}} \beta_{z, k}^j dz + \lambda^k w_k \int_{\mathbb{Z}} \beta_{z, k}^k dz + (1 - \lambda^j - \lambda^k) \int_{\mathbb{Z}} \beta_{z, k}^i dz &\geq \\ \lambda^j w_j \int_{\mathbb{Z}} \beta_{z, i}^j dz + \lambda^k w_k \int_{\mathbb{Z}} \beta_{z, i}^k dz + (1 - \lambda^j - \lambda^k) \int_{\mathbb{Z}} \beta_{z, i}^i dz &\geq \\ \lambda^j w_j \int_{\mathbb{Z}} \beta_{z, j}^j dz + \lambda^k w_k \int_{\mathbb{Z}} \beta_{z, j}^k dz + (1 - \lambda^j - \lambda^k) \int_{\mathbb{Z}} \beta_{z, j}^i dz.\end{aligned}\tag{29}$$

Notice that the first line in (29) equals the world spending on commodities produced in  $k$ , the second one equals the world spending on commodities produced in  $i$ , and the third one



equals the world spending on commodities produced in  $j$ . However, when  $w_k < 1 < w_j$ , those inequalities are inconsistent with market clearing conditions (9). As a result, there cannot exist an equilibrium with measure  $\lambda_j > 0$  of countries with  $w_j > 1$  and/or a measure  $\lambda_k > 0$  of countries with  $w_k < 1$ . ■

**Proof of Proposition 2.** Preliminarily, notice that (23) together with (24) yields:

$$\beta_{z',v'} = \frac{q_{z',v'}}{\int_{\mathbb{Z}} \int_{\mathbb{V}} q_{z'',v''} dv'' dz''}. \quad (30)$$

**Part (i).** From (19), together with Lemma 1 and Proposition 1, we have:

$$(\eta_{z,v} - 1) \ln q_{z,v} = \ln(1 + \kappa) - \ln \mu; \quad (31)$$

thus, computing (31) for any pair of commodities  $(z', v'), (z'', v'') \in \mathbb{Z} \times \mathbb{V}$  yields:

$$(\eta_{z',v'} - 1) \ln q_{z',v'} = (\eta_{z'',v''} - 1) \ln q_{z'',v''}. \quad (32)$$

Hence, (32) implies that  $q_{z',v'} > q_{z'',v''}$  when  $\eta_{z',v'} < \eta_{z'',v''}$ . By considering this result in conjunction with (30), our claim immediately follows.

**Part (ii).** Differentiating (32) with respect to  $\kappa$  yields:

$$\frac{dq_{z',v'}}{d\kappa} = \frac{\eta_{z'',v''} - 1}{\eta_{z',v'} - 1} \frac{q_{z',v'}}{q_{z'',v''}} \frac{dq_{z'',v''}}{d\kappa}. \quad (33)$$

Using (23), (31) and (33):

$$\frac{dq_{z',v'}}{d\kappa} = \frac{1}{1 + \kappa} \left[ \frac{\eta_{z',v'} - 1}{q_{z',v'}} - \frac{1}{\mu} \left( \int_{\mathbb{Z}} \int_{\mathbb{V}} \frac{\eta_{z'',v''} - 1}{\eta_{z'',v''} - 1} \frac{q_{z'',v''}}{q_{z',v'}} dv'' dz'' \right) \right]^{-1} > 0 \quad (34)$$

Furthermore, from (30), and considering (33) and (34):

$$\frac{d\beta_{z',v'}}{d\kappa} = \frac{1}{\mu^2} \frac{dq_{z',v'}}{d\kappa} \left( \int_{\mathbb{Z}} \int_{\mathbb{V}} \left( \frac{\eta_{z'',v''} - \eta_{z',v'}}{\eta_{z'',v''} - 1} \right) q_{z'',v''} dv'' dz'' \right) \quad (35)$$

It is then easy to observe that (33) implies that  $dq_{z',v'}/d\kappa > dq_{z'',v''}/d\kappa$  when  $\eta_{z',v'} < \eta_{z'',v''}$ . By considering this result in conjunction with (35), our claim immediately follows.

**Part iii).** From (23), it immediately follows that  $\int_{\mathbb{Z}} \int_{\mathbb{V}} (d\beta_{z,v}/d\kappa) dv dz = 0$ . Given our result in part (ii), there must exist a threshold:

$$\hat{\eta} \equiv \frac{\int_{\mathbb{Z}} \int_{\mathbb{V}} [\eta_{z,v} q_{z,v} / (\eta_{z,v} - 1)] dv dz}{\int_{\mathbb{Z}} \int_{\mathbb{V}} [q_{z,v} / (\eta_{z,v} - 1)] dv dz}$$

associated to a subset of commodities  $(\hat{z}, \hat{v}) \in \mathbb{Z} \times \mathbb{V}$  such that  $d\beta_{\hat{z}, \hat{v}}/d\kappa = 0$ . Then it immediately follows that, for any  $\eta_{z', v'} \leq \hat{\eta}$ :  $d\beta_{z', v'}/d\kappa \geq 0$ ; and for any  $\eta_{z'', v''} \geq \hat{\eta}$ :  $d\beta_{z'', v''}/d\kappa \leq 0$ . (With strict inequalities if  $\eta_{z', v'} < \hat{\eta} < \eta_{z'', v''}$ .) ■

**Proof of Proposition 3.** We prove the proposition in different steps. We first prove that, if an equilibrium exists, then for all  $h \in \mathcal{H}$  and all  $l \in \mathcal{L}$ , it must necessarily be the case that: 1)  $w_h \neq w_l$ ; 2)  $w_h = w_H$ ,  $w_l = w_L$ ; 3)  $w_H/w_L > 1$ ; 4)  $w_H/w_L < \infty$ . Lastly, we prove that a unique equilibrium exists, with: 5)  $1 < w_H/w_L < \infty$ .

Preliminarily, consider a generic country  $i \in \mathbb{V}$ , and compute the aggregate demand by  $i$  for goods produced in country  $v \in \mathbb{V}$ . From the first-order conditions, it follows that:

$$\beta_{z,v}^i = \max \left\{ \left[ \frac{(1+\kappa)(w_i/w_v)}{(\mu^i)^{\eta_{z,v}}} \right]^{\frac{1}{\eta_{z,v}-1}}, \frac{1}{\mu^i} \right\}. \quad (36)$$

Hence, total demand by  $i$  for goods produced in  $h \in \mathcal{H}$  and in  $l \in \mathcal{L}$  are given, respectively by:

$$\int_{\mathbb{Z}} \beta_{z,h}^i w_i dz = w_i \int_{\underline{\eta}}^{\bar{\eta}} \max \left\{ \left( \frac{(1+\kappa) w_i}{(\mu^i)^{\eta} w_h} \right)^{1/(\eta-1)}, \frac{1}{\mu^i} \right\} \frac{1}{\bar{\eta} - \underline{\eta}} d\eta, \quad \text{for any } h \in \mathcal{H}, \quad (37)$$

and

$$\int_{\mathbb{Z}} \beta_{z,l}^i w_i dz = w_i \max \left\{ \left( \frac{(1+\kappa) w_i}{(\mu^i)^{\bar{\eta}} w_l} \right)^{1/(\bar{\eta}-1)}, \frac{1}{\mu^i} \right\}, \quad \text{for any } l \in \mathcal{L}. \quad (38)$$

*Step 1.* Suppose now that, in equilibrium,  $w_v = w$  for all  $v \in \mathbb{V}$ . Recalling Lemma 1, in this case the demand intensities in (36) are such that  $\beta_{z,v}^i = \beta_{z,v} = \mu^{-\eta_{z,v}/(\eta_{z,v}-1)} (1+\kappa)^{1/(\eta_{z,v}-1)}$  for all  $i \in \mathbb{V}$ . As a result, the value in (37) is strictly larger than the value in (38), since  $(1+\kappa)^{1/(\eta-1)} / \mu^{\eta/(\eta-1)}$  is strictly decreasing in  $\eta$ . As a consequence, given that  $i$  represents a generic country in  $\mathbb{V}$ , integrating over the set  $\mathbb{V}$ , it follows that the world demand for goods produced in a country from  $\mathcal{H}$  will be strictly larger than the world demand for goods produced in a country from  $\mathcal{L}$ . But this is inconsistent with the market clearing conditions, which require that world demand is equal for all  $v \in \mathbb{V}$ . Hence,  $w_v = w$  for all  $v \in \mathbb{V}$  cannot hold in equilibrium.

*Step 2.* Suppose that, in equilibrium,  $w_{h'} > w_{h''}$  for some  $h', h'' \in \mathcal{H}$ . Computing (37) respectively for  $h'$  and  $h''$  yields:

$$w_i \int_{\underline{\eta}}^{\bar{\eta}} \max \left\{ \left( \frac{(1+\kappa) w_i}{(\mu^i)^{\eta} w_{h'}} \right)^{\frac{1}{\eta-1}}, \frac{1}{\mu^i} \right\} \frac{1}{\bar{\eta} - \underline{\eta}} d\eta \leq w_i \int_{\underline{\eta}}^{\bar{\eta}} \max \left\{ \left( \frac{(1+\kappa) w_i}{(\mu^i)^{\eta} w_{h''}} \right)^{\frac{1}{\eta-1}}, \frac{1}{\mu^i} \right\} \frac{1}{\bar{\eta} - \underline{\eta}} d\eta$$

Now, since  $i$  represents a generic country in  $\mathbb{V}$ , integrating over the set  $\mathbb{V}$ , it follows that the world demand for goods produced in country  $h'$  will be no larger than the world demand for

goods produced in country  $h''$ . But this is inconsistent with the market clearing conditions, which require that world demand for goods produced in country  $h'$  must be strictly larger than world demand for goods produced in country  $h''$ . Furthermore, an analogous reasoning rules out  $w_{h'} < w_{h''}$ . As a consequence, it must be the case that, if an equilibrium exists, it must be characterised by  $w_{h'} = w_{h''}$  for any  $h', h'' \in \mathcal{H}$ . (Similarly, it can be proved that, if an equilibrium exists, it must be characterised by  $w_{l'} = w_{l''}$  for any  $l', l'' \in \mathcal{L}$ .)

*Step 3.* Bearing in mind the result in the previous step, denote by  $w_L$  the wage of a country belonging to  $\mathcal{L}$  and by  $w_H$  the wage of a country belonging to  $\mathcal{H}$ . In addition, without any loss of generality, let  $w_L = 1$  (i.e., take  $w_L$  as the *numeraire* of the world economy). Suppose now that  $w_H < 1$ . Since  $[(1 + \kappa)(w_i/w_v) / (\mu^i)^\eta]^{1/(\eta-1)}$  is strictly decreasing in  $\eta$ , it follows that the value in (38) is no larger than the value in (37). Moreover, since  $i$  represents a generic country in  $\mathbb{V}$ , integrating over the set  $\mathbb{V}$ , we obtain that the world demand for goods produced in a country from region  $\mathcal{L}$  is no larger than world demand for goods produced in a country from region  $\mathcal{H}$ . But this is inconsistent with the market clearing conditions when  $w_H < 1$ , which require that world demand for goods produced in a country from region  $\mathcal{L}$  must be strictly larger than world demand for goods produced in a country from region  $\mathcal{H}$ .

*Step 4.* As a result of steps 1, 2 and 3, our only remaining candidate for an equilibrium is then  $w_H > w_L = 1$ . From (37), it follows that the aggregate demand by any  $h' \in \mathcal{H}$  for goods produced in region  $\mathcal{H}$  coincides with its aggregate supply to the same region. Hence, there must be no net surplus within region  $\mathcal{H}$ . Analogously, from (38) it follows that there must be no net surplus within region  $\mathcal{L}$ . As a result, a necessary condition for market clearing is that the aggregate demand by region  $\mathcal{L}$  for goods produced in region  $\mathcal{H}$  must equal the aggregate demand by region  $\mathcal{H}$  for goods produced in region  $\mathcal{L}$ . Formally:

$$\int_{\mathcal{L}} \int_{\mathcal{H}} \int_{\mathbb{Z}} \beta_{z,h}^{l'} w_{l'} dz dh dl' = \int_{\mathcal{H}} \int_{\mathcal{L}} \int_{\mathbb{Z}} \beta_{z,l}^{h'} w_{h'} dz dl dh' \quad (39)$$

Suppose now that  $w_H \rightarrow \infty$ . Then, on the one hand, from (37) we obtain the aggregate demand by  $l' \in \mathcal{L}$  for goods produced in region  $\mathcal{H}$  would be equal to a finite (non-negative) number. Since this would hold true for every  $l' \in \mathcal{L}$ , then the aggregate demand by region  $\mathcal{L}$  for goods produced in region  $\mathcal{H}$  —left-hand side of (39)— would be equal to a finite (non-negative) number. On the other hand, from (37) it follows that when  $w_H \rightarrow \infty$  the aggregate demand by  $h' \in \mathcal{H}$  for goods produced in any  $l \in \mathcal{L}$  would tend to infinity. Since this would hold true for every  $h' \in \mathcal{H}$  and  $l \in \mathcal{L}$ , then the aggregate demand by region  $\mathcal{H}$  for goods produced in region  $\mathcal{L}$  —right-hand side of (39)— would also tend to infinity. But this then is inconsistent with the equality required by

condition (39). Hence, if an equilibrium exists, it must be then characterised by  $w_L < w_H < \infty$ .

*Step 5.* Finally, we prove now that there exists an equilibrium  $1 < w_H < \infty$ , and this equilibrium is unique. Recall that, by setting  $w_L = 1$ ,  $w_H$  represents the relative wage between region  $\mathcal{H}$  and region  $\mathcal{L}$ . Step 1 shows that, should the relative wage equal one, then the world demand for goods produced in a country from  $\mathcal{H}$  would be strictly larger than the world demand for goods produced in a country from  $\mathcal{L}$ . Step 4 shows instead that, should  $w_H \rightarrow \infty$ , then the world demand for goods produced in a country from  $\mathcal{H}$  would be strictly smaller than the world demand for goods produced in a country from  $\mathcal{L}$ . Consider now (36) for any  $v = h \in \mathcal{H}$ , implying that  $w_h = w_H$ , and notice that the demand intensities  $\beta_{z,h}^i$  are all non-increasing in  $w_H$ . In addition, consider (36) for any  $v = l \in \mathcal{L}$ , implying that  $w_l = 1$ , and notice that in this case the  $\beta_{z,l}^i$  are all non-decreasing in  $w_H$ , while they are strictly increasing in  $w_H$  for at least some  $z \in \mathbb{Z}$  when  $i \in \mathcal{H}$ . Therefore, taking all this into account, together with the expressions in (37) and (38), it follows that the world demand for goods produced in a country from  $\mathcal{L}$  may increase with  $w_H$ , while world demand for goods produced in a country from  $\mathcal{H}$  will decrease with  $w_H$ . Hence, by continuity, there must necessarily exist some  $1 < w_H < \infty$  consistent with all market clearing conditions holding simultaneously. In addition, this equilibrium must then also be unique. ■

**Lemma 3** *Let  $w_l = w_L = 1$  for all  $l \in \mathcal{L}$ , and  $w_h = w_H > 1$  for all  $h \in \mathcal{H}$ . Then, for any country  $i \in \mathbb{V}$ ,  $q_{z,l}^i > 1$  for all  $(z, l) \in \mathbb{V} \times \mathcal{L}$ .*

**Proof.** Consider first a consumer from region  $\mathcal{H}$ . Then, (19) and (20) for goods produced in  $l \in \mathcal{L}$  lead to:

$$\delta_L^H = (\bar{\eta} - 1) \ln q_L^H - \ln w_H + \ln \mu^H - \ln(1 + \kappa), \quad \text{for all } (z, l) \in \mathbb{Z} \times \mathcal{L}; \quad (40)$$

whereas the same two conditions for goods produced in  $h \in \mathcal{H}$  lead to:

$$\delta_{z,h}^H = (\eta_{z,h} - 1) \ln q_{z,h}^H + \ln \mu^H - \ln(1 + \kappa), \quad \text{for all } (z, h) \in \mathbb{Z} \times \mathcal{H}. \quad (41)$$

Firstly, suppose that  $\delta_{z,v}^H > 0$  for all  $(z, v) \in \mathbb{Z} \times \mathbb{V}$ . Then  $q_{z,v}^H = 1$  always, which in turn implies  $\mu^H = 1$ . Replacing these values into (40) leads to  $\delta_L^H = -\ln w_H - \ln(1 + \kappa) < 0$ , which contradicts  $\delta_{z,l}^H > 0$ . Hence, it must then be  $\delta_{z,v}^H = 0$  for at least some  $(z, v) \in \mathbb{Z} \times \mathbb{V}$ . Secondly, let  $z', z'' \in \mathbb{Z}$  and  $l', l'' \in \mathcal{L}$ , and suppose that  $\delta_{z',l'}^H > \delta_{z'',l''}^H = 0$ . Then, from (40), it follows that  $(\bar{\eta} - 1) \ln q_{z'',l''}^H < 0$ , which is impossible. Hence, either (A)  $\delta_L^H > 0$  for all  $(z, l) \in \mathbb{Z} \times \mathcal{L}$ ; or (B)  $\delta_L^H = 0$  for all  $(z, l) \in \mathbb{Z} \times \mathcal{L}$ . Finally, suppose that  $\delta_L^H > \delta_{z,h}^H = 0$ . Then, from (40) and (41), it

follows that  $(\eta_{z,h} - 1) \ln q_{z,h}^H < 0$ , which is again impossible. Hence, (B) must necessarily hold. An analogous reasoning shows that  $\delta_L^L = 0$  for all  $(z, l) \in \mathbb{Z} \times \mathcal{L}$  holds for any consumer from  $\mathcal{L}$ . Hence, it must be that, for any country  $i \in \mathbb{V}$ ,  $q_{z,l}^i > 1$  for all  $(z, l) \in \mathbb{V} \times \mathcal{L}$ . ■

**Proof of Proposition 4.**

Preliminarily, note that Lemma 3 implies that  $\delta_L^L = \delta_L^H = 0$  (see lemma and proof above). Then, for a consumer in any country in region  $\mathcal{L}$ :

$$-(\bar{\eta} - 1) \ln q_L^L - \ln \mu^L + \ln(1 + \kappa) = 0, \text{ for all } (z, l) \in \mathbb{Z} \times \mathcal{L}; \quad (42)$$

$$-(\eta_{z,h} - 1) \ln q_{z,h}^L - \ln \mu^L + \ln(1 + \kappa) - \ln w_H + \delta_H^L = 0, \text{ for all } (z, h) \in \mathbb{Z} \times \mathcal{H}; \quad (43)$$

whereas, for a consumer in any country in region  $\mathcal{H}$ :

$$-(\bar{\eta} - 1) \ln q_L^H - \ln \mu^H + \ln(1 + \kappa) + \ln w_H = 0, \text{ for all } (z, l) \in \mathbb{Z} \times \mathcal{L}; \quad (44)$$

$$-(\eta_{z,h} - 1) \ln q_{z,h}^H - \ln \mu^H + \ln(1 + \kappa) + \delta_H^H = 0, \text{ for all } (z, h) \in \mathbb{Z} \times \mathcal{H}. \quad (45)$$

Using in turn (42), (43) and (46), (44), and (45) and (48), we then obtain:

$$q_L^L = [(1 + \kappa) / \mu^L]^{1/(\bar{\eta}-1)}, \quad \forall (z, l) \in \mathbb{Z} \times \mathcal{L}; \quad (46)$$

$$q_{z,h}^L = \left( (q_L^L)^{\bar{\eta}-1} e^{\delta_H^L} / w_H \right)^{1/(\eta_{z,h}-1)} = \left( (1 + \kappa) e^{\delta_H^L} / (\mu^L w_H) \right)^{1/(\eta_{z,h}-1)}, \quad \forall (z, h) \in \mathbb{Z} \times \mathcal{H}; \quad (47)$$

$$q_L^H = [(1 + \kappa) w_H / \mu^H]^{1/(\bar{\eta}-1)}, \quad \forall (z, l) \in \mathbb{Z} \times \mathcal{L}; \quad (48)$$

$$q_{z,h}^H = \left( (q_L^H)^{(\bar{\eta}-1)} e^{\delta_H^H} / w_H \right)^{1/(\eta_{z,h}-1)} = \left( (1 + \kappa) e^{\delta_H^H} / \mu^H \right)^{1/(\eta_{z,h}-1)}, \quad \forall (z, h) \in \mathbb{Z} \times \mathcal{H}. \quad (49)$$

**Part (i).** Firstly, suppose that  $q_L^H \leq q_L^L$ . From (46) and (48), it follows that  $1 < \mu^L < w_H \mu^L \leq \mu^H$ , which together with the definitions of  $\mu^H$  and  $\mu^L$  imply  $q_{z,h}^H > q_{z,h}^L \geq 1$  at least for some  $(z, h)$  and, from (47) and (49), for all  $(z, h) \in \mathbb{Z} \times \mathcal{H}$ . From (47) and (49), this in turn entails  $w_H \mu^L e^{-\delta_H^L} > \mu^H$ , which leads to a contradiction since  $w_H \mu^L e^{-\delta_H^L} \leq w_H \mu^L$ . Hence, it must be that  $q_L^H > q_L^L$ .

Secondly, suppose that  $q_{z,h}^H = 1$ . From (49), it follows that  $\mu^H = (1 + \kappa) e^{\delta_H^H}$ . Since  $q_L^H > q_L^L$ , then, from (46) and (48),  $w_H \mu^L > \mu^H$ . The last two expressions entail  $w_H \mu^L > \mu^H = (1 + \kappa) e^{\delta_H^H} > 1 + \kappa$ . Additionally, notice that  $q_{z,h}^L \geq 1$  in (47) implies  $e^{\delta_H^L} \geq \mu^L w_H / (1 + \kappa)$ . Then, the last two inequalities imply  $e^{\delta_H^L} \geq \mu^L w_H / (1 + \kappa) > 1$  and thus  $q_{z,h}^L = 1$ . Together with the fact that  $q_L^H > q_L^L > 1$ , this finally entails that  $\beta_L^H > \beta_H^H$  and  $\beta_L^L > \beta_H^L$ , which is actually inconsistent with market clearing conditions when  $w_H > 1$ . Hence, it must be that  $q_{z,h}^H > 1$ .

Finally, suppose that  $q_{z,h}^L = 1$ . From the definition of  $\mu^L$ , it follows that  $\mu^L = \lambda + (1 - \lambda) q_L^L$ .

From the market clearing condition for a country in  $\mathcal{L}$ , we have  $\lambda q_L^H w_H / \mu^H + (1 - \lambda) q_L^L / \mu^L = 1$ . The last two expressions lead to  $q_L^H w_H \mu^L = \mu^H$ , which in turn implies  $q_L^H < 1$  since  $w_H \mu^L > \mu^H$ , leading to a contradiction. Hence, it must be that  $q_{z,h}^L > 1$ . Furthermore, given that  $w_H \mu^L > \mu^H$ , from (47) and (49) it immediately follows that  $q_{z,h}^H > q_{z,h}^L$ . Thus, in equilibrium,  $q_{z,v}^H > q_{z,v}^L > 1$ , for all  $(z, v) \in \mathbb{Z} \times \mathbb{V}$ .

**Part (ii).** The claim straightforwardly follows from (43) and (49) by noting that  $\partial q_{z,h}^i / \partial \eta_{z,h} = -q_{z,h}^i \ln q_{z,h}^i / (\eta_{z,h} - 1) < 0$ , for any  $i \in \mathbb{V}$ .

**Part (iii).** When imposing  $q_{z,h}^L = q_L^L$ , (42) and (43) yield  $(\bar{\eta} - 1) \ln q_L^L = (\eta_{z,h} - 1) \ln q_L^L + \ln w_H$ . Isolating  $\eta_{z,h}$  we then have  $\eta_{z,h} = \bar{\eta} - \ln w_H / \ln q_L^L \equiv \eta' < \bar{\eta}$ . Suppose now that  $\eta' \leq \underline{\eta}$ . Since  $\partial q_{z,h}^L / \partial \eta_{z,h} < 0$ , from the definition of  $\eta'$  it follows that  $q_{z,h}^L \leq q_L^L$  for all  $(z, h) \in \mathbb{Z} \times \mathcal{H}$ , with strict inequalities for all  $(z, h)$  such that  $\eta_{z,h} > \underline{\eta}$ . From the definition of  $\mu^L$ , we thus obtain that  $\mu^L < q_L^L$ . From the market clearing condition (9), taken for a country in region  $\mathcal{L}$ , we get  $1 - \lambda q_L^H w_H / \mu^H = (1 - \lambda) q_L^L / \mu^L > 1 - \lambda$ , which in turn implies that  $q_L^H w_H / \mu^H < 1$ . Together with  $w_H \mu^L > \mu^H$  and  $\mu^L < q_L^L$ , the last inequality finally yields  $q_L^H < \mu^L < q_L^L$ , leading to a contradiction. Hence, it must be that  $\eta' > \underline{\eta}$ . Thus, given the definitions of  $q_{z,\bar{\eta}}^i$  and  $q_{z,\underline{\eta}}^i$ , in equilibrium  $q_{z,\bar{\eta}}^i < q_L^i < q_{z,\underline{\eta}}^i$ . ■

**Proof of Proposition 5.** The proof follows straightforwardly from noting that both  $\beta_{z,h}^H$  and  $\beta_{z,h}^L$  in (15) are functions of  $\eta_{z,h}$ , and part (ii) of Proposition 4 implies that  $\partial \beta_{z,h}^H / \partial \eta_{z,h} < 0$  and  $\partial \beta_{z,h}^L / \partial \eta_{z,h} < 0$ . ■

**Proof of Proposition 6.** Consider a consumer from region  $\mathcal{H}$ . (19) and (20) imply that:

$$\begin{aligned} \ln(1 + \kappa) &= (\eta_{z,h} - 1) \ln \beta_{z,h}^H + \eta_{z,h} \ln \mu^H, \quad \text{for all } (z, h) \in \mathbb{Z} \times \mathcal{H} \\ &= (\bar{\eta} - 1) \ln \beta_{z,l}^H + \bar{\eta} \ln \mu^H - \ln w_H, \quad \text{for all } (z, l) \in \mathbb{Z} \times \mathcal{L}. \end{aligned}$$

Similarly, considering a consumer from region  $\mathcal{L}$ , first-order conditions will imply:

$$\begin{aligned} \ln(1 + \kappa) &= (\eta_{z,h} - 1) \ln \beta_{z,h}^L + \eta_{z,h} \ln \mu^L + \ln w_H, \quad \text{for all } (z, h) \in \mathbb{Z} \times \mathcal{H} \\ &= (\bar{\eta} - 1) \ln \beta_{z,l}^L + \bar{\eta} \ln \mu^L, \quad \text{for all } (z, l) \in \mathbb{Z} \times \mathcal{L}. \end{aligned}$$

From these expressions, after some simple algebra, we may obtain the following relation:

$$(\bar{\eta} - \eta_{z,h}) \ln \left( \frac{\mu^H}{\mu^L} \right) = (\eta_{z,h} - 1) \ln \left( \frac{\beta_{z,h}^H}{\beta_{z,h}^L} \right) + (\bar{\eta} - 1) \ln \left( \frac{\beta_{z,l}^L}{\beta_{z,l}^H} \right). \quad (50)$$

Notice then that, since  $\ln(\mu^H / \mu^L) > 0$ , from (50) we may obtain that:

$$\begin{aligned} \beta_{z,h}^H / \beta_{z,h}^L &> \beta_{z,l}^H / \beta_{z,l}^L, \quad \text{whenever } \eta_{z,h} < \bar{\eta}; \\ \beta_{z,h}^H / \beta_{z,h}^L &= \beta_{z,l}^H / \beta_{z,l}^L \quad \text{when } \eta_{z,h} = \bar{\eta}. \end{aligned} \quad (51)$$

In addition, from (50) we may also obtain that, for any two pairs  $(z, h')$  and  $(z, h'')$  such that  $\eta_{z, h'} < \eta_{z, h''}$ ,  $\beta_{z, h'}^H / \beta_{z, h'}^L > \beta_{z, h''}^H / \beta_{z, h''}^L$ . Hence, letting  $\underline{\eta} < \eta_{z, h'} < \eta_{z, h''} = \bar{\eta}$ , an immediate implication of (51) is that

$$\frac{\beta_{z, \underline{\eta}}^H}{\beta_{z, \underline{\eta}}^L} > \frac{\beta_{z, \eta_{z, h'}}^H}{\beta_{z, \eta_{z, h'}}^L} > \frac{\beta_{z, \eta_{z, h''}}^H}{\beta_{z, \eta_{z, h''}}^L} = \frac{\beta_{z, \bar{\eta}}^H}{\beta_{z, \bar{\eta}}^L}. \quad (52)$$

Now, suppose that  $\beta_{z, \underline{\eta}}^H \leq \beta_{z, \underline{\eta}}^L$ . Then, from (52) it follows that  $\beta_{z, v}^H < \beta_{z, v}^L$  for all  $(z, v) \in \mathbb{Z} \times \mathbb{V}$ . However, since in the optimum the budget constraint of a consumer from  $\mathcal{L}$  must hold with equality, then the fact that  $\beta_{z, \underline{\eta}}^H \leq \beta_{z, \underline{\eta}}^L$  would in turn imply that  $\int_{\mathbb{Z}} \int_{\mathbb{V}} \beta_{z, v}^H dv dz < \int_{\mathbb{Z}} \int_{\mathbb{V}} \beta_{z, v}^L dv dz = 1$ . But,  $\int_{\mathbb{Z}} \int_{\mathbb{V}} \beta_{z, v}^H dv dz < 1$  is inconsistent with consumers maximising behaviour in  $\mathcal{H}$ . As a consequence, it must thus be the case that  $\beta_{z, \underline{\eta}}^H > \beta_{z, \underline{\eta}}^L$ . ■

## Appendix B: Additional theoretical results

**Proposition 7** Suppose that the set  $\mathbb{V}$  is composed by  $K$  disjoint subsets, indexed by  $k = 1, \dots, K$ , each denoted by  $\mathcal{V}_k \subset \mathbb{V}$  and with Lebesgue measure  $\lambda_k > 0$ . Assume that, for any country  $v_k \in \mathcal{V}_k$ : (a) in  $(\eta_k - \underline{\eta}) / (\bar{\eta} - \underline{\eta})$  sectors,  $\eta_{z,v_k} = \eta_k$  (with  $\eta_k$  such that: for any  $k' < k''$ ,  $\eta_{k'} < \eta_{k''}$ ; for  $k = 1$ ,  $\eta_k = \underline{\eta}$ ; for  $k = K$ ,  $\eta_k = \bar{\eta}$ ); (b) in the remaining  $(\bar{\eta} - \eta_k) / (\bar{\eta} - \underline{\eta})$  sectors,  $\eta_{z,v_k}$  is independently drawn from a uniform distribution with support  $[\eta_k, \bar{\eta}]$ . Then:  $w_1 > \dots > w_{k'} > \dots > w_K$ , where  $1 < k' < K$ .

**Proof.** Combining (19) and (20), and recalling that  $\delta_{z,v}^i = 0$ , for all  $(z, v) \in \mathbb{Z} \times \mathbb{V}$ , yields:

$$\beta_{z,v}^i = \left[ (1 + \kappa) \left( \frac{w_i}{w_v} \right) (\mu^i)^{-\eta_{z,v}} \right]^{1/(\eta_{z,v}-1)} \equiv \beta^i(\eta_{z,v}, w_v). \quad (53)$$

Notice from (53) that  $\partial \beta^i(\eta_{z,v}, w_v) / \partial \eta_{z,v} < 0$  and  $\partial \beta^i(\eta_{z,v}, w_v) / \partial w_v < 0$ .

Consider two generic regions  $k' < k''$ , and suppose that  $w_{k'} \leq w_{k''}$ . Since  $\beta_{z,v}^i$  is strictly decreasing in  $\eta$  and  $w_v$ , (53) holds for all  $z \in \mathbb{Z}$ , and by the law of large numbers and the fact that the distribution of  $\eta_{z,k'}$  FOSD the distribution of  $\eta_{z,k''}$ , it follows that  $\int_{\mathbb{Z}} \beta_{z,k'}^i dz > \int_{\mathbb{Z}} \beta_{z,k''}^i dz$ . Moreover, since  $i$  represents a generic country in  $\mathbb{V}$ , integrating over the set  $\mathbb{V}$ , we obtain that the world demand for goods produced in a country from region  $k'$  is larger than world demand for goods produced in a country from region  $k''$ . But this is inconsistent with the market clearing conditions when  $w_{k'} \leq w_{k''}$ , which require that world demand for goods produced in a country from region  $k'$  must be no larger than world demand for goods produced in a country from region  $k''$ . As a consequence, it must be that  $w_{k'} > w_{k''}$ . ■

**Proposition 8** For country  $v_1 \in \mathcal{V}_1$  such that  $\eta_{z,v_1} = \underline{\eta}$  and any country  $v_k \in \mathcal{V}_k$  such that  $\eta_{z,v_k} = \eta_k$ :  $RCA_{z,v_1} > RCA_{z,v_k}$ , for any  $z \in \mathbb{Z}$ .

**Proof.** Countries with identical incomes have identical budget shares. Let  $\beta_{z,v}^j$  denote the common budget share for  $(z, v)$  in  $j$ . Then, from the definition of total value of good  $z$  by country  $v$ , we have that  $X_{z,v} = \sum_{j=1}^K \lambda_j \beta^j(\eta_{z,v}, w_v) w_j$ . Notice also that  $X_v = w_v$  and  $W_z/W = 1$ . Hence, (10) yields:

$$RCA_{z,v} = \frac{\sum_{j=1}^K \lambda_j \beta^j(\eta_{z,v}, w_v) w_j}{w_v}. \quad (54)$$

Consider a generic good  $z \in \mathbb{Z}$  and, without loss of generality, select countries:  $v_1 \in \mathcal{V}_1$  such that  $\eta_{z,v_1} = \underline{\eta}$ ; and  $v_k \in \mathcal{V}_k$  from any region  $k \in (1, K]$  such that  $\eta_{z,v_k} = \eta_k$ . From (54) we



obtain that  $RC A_{z,v_1} > RC A_{z,v_k}$  requires:

$$\frac{\sum_{j=1}^K \lambda_j \beta^j(\underline{\eta}, w_1) w_j}{\sum_{j=1}^K \lambda_j \beta^j(\eta_k, w_k) w_j} > \frac{w_1}{w_k}. \quad (55)$$

Compute (9) respectively for  $v_1$  and  $v_k$ , transform the integrals over  $z$  in integrals in  $\eta$ , divide side by side the former by the latter, and rearrange to obtain:

$$\begin{aligned} \int_{\underline{\eta}}^{\eta_k} \sum_{j=1}^K \lambda_j \beta^j(\eta, w_1) w_j d\eta &= \frac{w_1}{w_k} (\eta_k - \underline{\eta}) \sum_{j=1}^K \lambda_j \beta^j(\eta_k, w_k) w_j + \\ &\quad \int_{\eta_k}^{\bar{\eta}} \sum_{j=1}^K \lambda_j \left[ \frac{w_1}{w_k} \beta^j(\eta, w_k) - \beta^j(\eta, w_1) \right] w_j d\eta. \end{aligned}$$

Since  $w_1 > w_k$ ,  $\beta^j(\eta, w_1) < \beta^j(\eta, w_k)$  for any given  $\eta$ . Hence, the second term on the RHS of last expression is positive. This leads to:

$$\int_{\underline{\eta}}^{\eta_k} \sum_{j=1}^K \lambda_j \beta^j(\eta, w_1) w_j d\eta > \frac{w_1}{w_k} (\eta_k - \underline{\eta}) \sum_{j=1}^K \lambda_j \beta^j(\eta_k, w_k) w_j. \quad (56)$$

Furthermore, since  $\beta^j(\eta, w_1) < \beta^j(\underline{\eta}, w_1)$  for all  $\eta > \underline{\eta}$ , it follows that:

$$\int_{\underline{\eta}}^{\eta_k} \sum_{j=1}^K \lambda_j \beta^j(\eta, w_1) w_j d\eta < (\eta_k - \underline{\eta}) \sum_{j=1}^K \lambda_j \beta^j(\underline{\eta}, w_1) w_j.$$

Combining this last result with (56), and simplifying, we finally get:

$$\sum_{j=1}^K \lambda_j \beta^j(\underline{\eta}, w_1) w_j > \frac{w_1}{w_k} \sum_{j=1}^K \lambda_j \beta^j(\eta_k, w_k) w_j,$$

which straightforwardly implies that (55) holds for any  $\eta_k > \underline{\eta}$ . ■

**Proposition 9** Let  $\beta_{z,\underline{\eta}}^j$  denote the demand intensity by a consumer from country  $j \in \mathcal{V}_j$  for the variety of good  $z$  produced in country  $v_1$  such that  $\eta_{z,v_1} = \underline{\eta}$ . Then:  $\beta_{z,\underline{\eta}}^1 > \dots > \beta_{z,\underline{\eta}}^{j'} > \dots > \beta_{z,\underline{\eta}}^K$ , where  $1 < j' < K$

**Proof.** Consider a pair of generic consumers from regions  $j'$  and  $j''$ , where  $j' < j''$ . In addition, consider a pair of generic exporters from countries  $v_{k'}$  and  $v_{k''}$ , where  $k' \leq k''$ . Following an analogous procedure as in the proof of Proposition 6, combining (19) and (20) of consumers  $j'$  and  $j''$  for the varieties of good  $z$  produced in  $v_{k'}$  and  $v_{k''}$ , we may obtain:

$$\left( \eta_{z,v_{k''}} - \eta_{z,v_{k'}} \right) \ln \left( \frac{\mu^{j'}}{\mu^{j''}} \right) = \left( \eta_{z,v_{k'}} - 1 \right) \ln \left( \frac{\beta_{z,v_{k'}}^{j'}}{\beta_{z,v_{k'}}^{j''}} \right) + \left( \eta_{z,v_{k''}} - 1 \right) \ln \left( \frac{\beta_{z,v_{k''}}^{j''}}{\beta_{z,v_{k''}}^{j'}} \right). \quad (57)$$

Since  $\ln(\mu^{j'}/\mu^{j''}) > 0$ , from (57) it follows that  $\beta_{z,v_{k'}}^{j'}/\beta_{z,v_{k'}}^{j''} \geq \beta_{z,v_{k''}}^{j'}/\beta_{z,v_{k''}}^{j''}$  when  $\eta_{z,v_{k'}} \leq \eta_{z,v_{k''}}$ . Now, let  $k' = 1$  and pick  $z$  such that  $\eta_{z,v_1} = \underline{\eta}$ . Next, suppose  $\beta_{z,\underline{\eta}}^{j'} \leq \beta_{z,\underline{\eta}}^{j''}$ . Then, we must have that  $\beta_{z,v}^{j'} \leq \beta_{z,v}^{j''}$  for all  $(z,v) \in \mathbb{Z} \times \mathbb{V}$ , with strict inequality for all  $(z,v)$  such that  $\eta_{z,v_k} > \underline{\eta}$ . However, since the budget constraints of consumer  $j'$  and  $j''$  require that  $\int_{\mathbb{Z}} \int_{\mathbb{V}} \beta_{z,v}^{j'} dv dz = \int_{\mathbb{Z}} \int_{\mathbb{V}} \beta_{z,v}^{j''} dv dz$ , then  $\beta_{z,\underline{\eta}}^{j'} \leq \beta_{z,\underline{\eta}}^{j''}$  cannot possibly be true. ■

## Appendix C: Additional empirical results

**Table 2.A (extended)**

	Dependent Variable: log impo shares of product $i$ from exporter $x$				
	full sample (1)	full sample (2)	excl. OECD (3)	excl. high income (4)	restr. sample (5)
Log RCA exporter	-0.469*** (0.106)	-0.293*** (0.107)	-0.066 (0.104)	-0.184 (0.132)	-0.330*** (0.098)
Interaction term	0.104*** (0.012)	0.091*** (0.012)	0.064*** (0.012)	0.078*** (0.016)	0.082*** (0.011)
Gravity Terms	Yes	Yes	Yes	Yes	Yes
Product dummies	Yes	-	-	-	-
Importer dummies	Yes	-	-	-	-
Exporter dummies	Yes	Yes	Yes	Yes	Yes
Importer-Product dummies	-	Yes	Yes	Yes	Yes
Observations	5,571,567	5,773,873	3,921,408	3,397,049	2,709,459
R squared	0.53	0.57	0.52	0.52	0.54

Robust absolute standard errors clustered at the importer and exporter level reported in parentheses. All data corresponds to the year 2009.

All regressions include: distance, contiguity, common official language, common coloniser, common legal origin and common currency.

Column (5) uses importers in *subset A* to compute the exporters' RCA and importers in *subset B* to compute the dependent variable.

\* significant 10%; \*\* significant 5%; \*\*\* significant 1%.

Subset A				Subset B			
country	GDP pc	country	GDP pc	country	GDP pc	country	GDP pc
Un. Arab Emirates	52855	Albania	6641	Luxembourg	84572	Turkmenistan	6936
Macau	51111	Samoa	6547	Bermuda	52091	Dominica	6580
Singapore	47313	Ukraine	6415	Norway	49974	Vanuatu	6531
Brunei	46206	Tunisia	6300	Kuwait	46747	El salvador	6341
USA	41147	Ecuador	6171	Australia	41288	Guatemala	6288
Switzerland	39632	Armenia	5376	Netherlands	40574	Algeria	6074
Iceland	37212	Egypt	4957	Austria	37413	Georgia	5063
Canada	36234	Namibia	4737	Sweden	35246	Angola	4756
Belgium	34625	Jordan	4646	Denmark	33929	Iraq	4709
United kingdom	33410	Maldives	4461	Ireland	33406	Bhutan	4566
Finland	32186	Fiji	4284	Japan	31980	Guyana	4336
Trinidad and Tobago	31057	Indonesia	4074	France	30837	Kiribati	4092
New Zealand	27878	Syrian Arab Rep.	3995	Bahamas	28382	Sri lanka	4034
Spain	27647	Cape verde	3770	Italy	27709	Bolivia	3792
Israel	25559	Honduras	3608	Greece	27305	Paraguay	3702
Slovenia	24956	Micronesia	3329	Korea, Rep.	25048	Swaziland	3444
Puerto rico	23664	India	3238	Seychelles	23864	Morocco	3292
Czech republic	23059	Viet nam	2871	Bahrain	23538	Mongolia	3170
Equatorial guinea	22008	Papua new guinea	2753	Barbados	22928	Philippines	2839
Saudi arabia	21542	Moldova	2496	Malta	21668	Lao	2636
Slovakia	19986	Uzbekistan	2384	Oman	20541	Yemen	2401
Libya	19233	Kyrgyzstan	2300	Portugal	19904	Pakistan	2353
Hungary	16521	Nicaragua	2190	Cyprus	18998	Congo	2223
Estonia	16294	Djibouti	2061	Poland	16376	Sudan	2188
Antigua-Barbuda	15047	Solomon islands	2004	Croatia	15084	Nigeria	2034
Russian federation	14645	Cameroon	1807	Palau	14988	Tajikistan	1873
Saint lucia	13079	Zambia	1765	Lithuania	14189	Cambodia	1768
Belarus	12782	Mauritania	1578	Lebanon	12907	Sao Tome-Princ.	1681
Saint Kitts-Nevis	12755	Gambia	1464	Latvia	12777	Senegal	1492
Chile	12007	Bangladesh	1397	Grenada	12024	Haiti	1444
Kazakhstan	11733	Lesotho	1309	Argentina	11960	Cote d'ivoire	1344
Cuba	11518	Ghana	1241	Mexico	11634	Chad	1276
Costa rica	11227	Kenya	1205	Malaysia	11309	Nepal	1209
Bulgaria	10923	Afghanistan	1171	Uruguay	11067	Tanzania,	1189
Iran	10620	Benin	1116	Suriname	10644	Uganda	1152
Panama	10187	Mali	999	Gabon	10276	Rwanda	1030
Dominican republic	9919	Burkina faso	900	Turkey	9920	Comoros	915
Azerbaijan	9619	Guinea	823	Romania	9742	Sierra leone	871
Brazil	9356	Mozambique	759	Mauritius	9487	Guinea-bissau	818
Botswana	8868	Togo	733	Venezuela	9123	Madagascar	753
Belize	8444	Malawi	652	Jamaica	8801	Ethiopia	684
Thailand	7799	Eritrea	593	Tonga	7862	Central Afr. Rep.	647
Saint Vincent-Gren.	7378	Somalia	461	Macedonia	7682	Niger	534
Bosnia-Herz.	7117	Burundi	368	Colombia	7528	Liberia	397
China	7008	Zimbabwe	143	Peru	7279	Congo, Dem. Rep.	231

GDP pc	Subset A	Subset B
Mean	12302	12931
Median	7063	7185
Max	52855	84572
Min	143	231
Std. Dev.	13315	14954

Note: we dropped Qatar from the sample whose GDP per head in 2009 was 159,144.

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